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Thermal Diffusion and Diffusion-Thermo Effects on Ohmic Heating with Mhd Free Convective Flow Past a Vertical Porous Plate in the Presence of Viscous Dissipation

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Abstract

The MHD laminar boundary layer flow with heat and mass transfer of an electrically conducting viscous fluid in a porous medium with ohmic heating and viscous dissipation effects is studied numerically. The governing systems of nonlinear partial differential equations of the flow, heat and mass transfer processes are solved using implicit finite difference scheme of Crank-Nicolson type. The numerical solutions are used to carry out parametric studies. The effects of the governing parameters on dimensionless quantities namely velocity, temperature, concentration, skin friction, local Nusselt and Sherwood numbers are explored. It is found that the dimensionless velocity and temperature increases with increase in the heating term and the concentration increases with suction and viscous dissipation.

1. Introduction

Two important phenomena in heat and mass transfer processes are thermal diffusion and diffusion-thermo effects. When heat and mass transfer occur simultaneously in a moving fluid affecting each other it causes a cross diffusion effect. It has been observed that an energy flux can be obtained not only by temperature gradients but also by concentration as reported by [1]. The mass flux caused by temperature gradient is called the thermal diffusion (Soret) effects, while the heat flux caused by concentration gradient is called the diffusion thermo (Dufour) effects.Thermal diffusion also known as Soret effect or thermophoresis is the diffusion of mass due to temperature gradient while diffusion thermo also known as Dufour effect is the transfer of heat generated by a concentration gradient.

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It has several industrial applications such as the optimum oil recovery from hydrocarbon reservoirs, fabrication of semi-conductor devices in molten metals and semi conductor mixtures, separation of species such as polymers, manipulation of macromolecules such as deoxyribonucleic acid (DNA) etc. It is also realized in the atmospheric and oceanic stream at the bottom of the oceans. Thermal di usion (Soret) e ect is important where more than one chemical species are present under very large temperature gradients, such as chemical reaction and isotope separation and in mixture of gases with very light molecular weight such as Hydrogen or Helium.

Due to the immense importance of thermal di usion and di usion thermo e ects, many researchers have ventured into the eld in order to address the various problems associated with its study and applications. Among such works are; [2] studied the in uence of Soret and Dufour e ects on MHD free convective heat and mass transfer ow over a vertical channel with constant suction and viscous dissipation. Their results revealed that velocity and temperature increase with increase in the viscous dissipation parameter and Dufour number, while an increase in Soret number causes a reduction in temperature and a rise in the velocity and concentration. [3] studied the thermal di usion and di usion thermo e ects on free convection heat and mass transfer from vertical surface in a porous medium with viscous dissipation in the presence on thermal radiation. [4] who studied the thermal di usion and di usion thermo e ects on unsteady MHD uid ow past a moving vertical plate embedded in a porous medium in the presence of hall current and rotating system. Similarly, [5] studied the e ects of thermal radiation, Soret and Dufour on an unsteady heat and mass transfer ow of a chemically reacting uid past a semi-in nite vertical plate with viscous dissipation. Their result shows that an increase in Eckert number of the uid actually increases the velocity and temperature pro les of the ow. Whereas an increase in thermal radiation parameter reduces the temperature distribution when the plate is being cooled. [6] studied the thermal di usion and di usion thermo e ects on ow of second grade uid between two inclined plane walls. [7] studied the di usion thermo and thermal di usion e ects with inclined magnetic eld on unsteady MHD slip ow over a permeable vertical plate. They found out that both di usion thermo and thermal di usion have the tendency of enhancing velocity pro les. [8] studied the thermal di usion and di usion thermo e ects on Casson nano uid ow in the presence of chemical reaction with inclined magnetic eld. [9] studied the thermal di usion and di usion thermo impacts on bio-convection Walter - B nanomaterial involving gyrotactic micro-organisms. They deduced that velocity intensi es with increase in melting and visco-elastic parameters while it decay against magnetic parameter. Also, micro-organism eld increases with Peclet number while it decays with bio-convection Lewis number among other points. [10] studied the thermal-di usion and di usion thermo e ects on heat and mass transfer in chemically reacting MHD Casson nano uid with viscous dissipation. They deduced that by raising the Casson parameter close to in nity, the behavior of casson uid obeys the law of viscosity. Conversion of energy via the work done by the uid molecules, in uences both dimensionless velocity and temperature pro les signi cantly, while the mass ux hikes the concentration pro le. The heat generated by the intermolecular reaction of uid particles resulted in a large amount of heat produced in the ow eld. [11] studied the role of di usion thermo on unsteady natural convection of a chemically reactive uid impacted by heat source/sink in a tube. The study revealed that the Dufour number elevates the Nusselt number and the skin friction. Furthermore, it posits that when the Dufour number is small, the temperature heightens with time , and then switches over to decreasing with time when the Dufour number is large. The study nds application in the pharmaceutical, petroleum and paint-making industries.

Magnetohydrodynamics (MHD) also called magneto-fluid dynamics or hydromagnetics is the study of the magnetic properties and behaviour of electrically conducting fluids. Examples of such magnetofluids include plasmas, liquid metals, salt water and electrolytes. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarize the fluid and reciprocally changes the magnetic field itself. MHD has several applications in the field of Geophysics, Astrophysics, Engineering, magnetic drug targeting, precision measurements through the use of sensors e.t.c. It is due to this immense importance of MHD that researchers have ventured into its study. Among such studies are the works of; [12] studied the MHD heat and mass transfer of chemical reaction fluid flow over a moving vertical plate in presence of heat source with convective surface boundary condition. They deduced that the thermal boundary layer thickness increases with the increase of source, chemical reaction parameter, plate surface convective heat parameter, and Schmidt number while the mass flux boundary layer thickness decreases. Moreover, the thermal boundary layer thickness, the mass boundary layer and velocity decrease as the Prandtl number increases. [13] studied the chemical reaction on MHD flow and heat transfer of a nanofluid near the stagnation point over a permeable stretching surface with non-uniform heat source/sink. They observed that heat source/sink with thermophoresis particle deposition in the presence of magnetic field have a substantial effect on the heat and mass transfer rate from the sheet to the fluid. As the strength of the chemical reaction is higher than the thermophoresis particle deposition, nanoparticle volume fraction of the fluid gradually decreases. [14] studied the chemical reaction and radiation effects on mixed convection heat and mass transfer over a vertical plate in power-law fluid saturated porous medium. They made the following conclusions; higher values of the radiation parameter R result in higher velocity, temperature distributions and non-dimensional mass transfer rate but lower concentration distribution and non-dimensional heat transfer rate. An increase in chemical reaction parameter, decrease in the velocity, concentration and heat transfer rate accompanied by an increase in temperature and mass transfer rate. An increase in power-law index parameter, increase in the velocity, but decrease in the temperature, heat and mass transfer rates within the boundary layer. [15] studied the heat and mass transfer on MHD flow of nanoparticles. [16] carried out a review on the numerical studies and applications of MHD in biological systems. The results indicated that during a surgery when it is necessary to drop blood flow or reduce tissue temperature, it may be achieved by using a magnetic field. Moreover, the review showed that the trapping is an important phenomenon in peristaltic flow that causes the formation of thrombus in blood and the movement of food bolus in gastrointestinal tract. [17] who studied the numerical analysis of MHD flows. [18] studied the heat and mass transfer of MHD for an unsteady viscous oscillatory flow. They deduced that diffusivity ratio leads to change in velocity and temperature profile among other points. [19] carried out an analytical investigation of MHD non- Newtonian type Casson nanofluid flow past a porous channel with periodic body acceleration. The result revealed that adding the nanoparticle has enhanced the velocity profile of the base fluid. Moreover, an increase in the periodic body acceleration results in enlarging velocity and temperature.

The application of ohmic heating was first introduced by [20] in late 1920's as a successful commercial technique referred to as 'electro-pure' process. But over the last few decades, a number of attempts have been made to use this technology in several food processing Industries and some industrial applications have resulted, including pasteurization of fruit products, evaporation, dehydration, fermentation and sterilization among others.

Since then, ohmic heating has emerged as an advanced heating method with diverse applications in food and agrochemical industries. The concept of ohmic heating in food processing is not new. Ohmic heating in food processing Industries can be found in the work of [21] who studied the electrical conductivities of selected solid foods during ohmic heating. [?] studied the extraction of soy milk using ohmic heating. [23] studied targeted genome editing in human cells with zinc nger nucleases' constructed via modular assembly. [24] studied ohmic heating in thermal technologies in food processing. [25] studied the electrical conductivity measurements and ascorbic acid degradation kinetics of strawberry products using ohmic heating. [26] studied a novel technique for minimization of electrochemical reactions during processing - pulsed ohmic heating. [27] compared the chemical properties of food products processed by conventional heating and ohmic heating. [28] studied the thermal characteristics of ohmically heated rice starch and rice ours. [29] studied the e ect of buoyancy force in computational uid dynamics simulation of a two-dimensional continuous ohmic heating process. [30] modelled ohmic heating patterns of multiphase food products using computational uid dynamics codes. [31] studied the mass transfer e ects on MHD mixed convective ow from a vertical surface with ohmic heating and viscous dissipation. [32] carried out a research on the in uence of ohmic heating/ osmotic dehydration treatments on polyphenoloxidase inactivation, physical properties and microbial stability of apples. [33] studied ohmic heating technology in food processing. [34] analysed the in uence of process parameters on mathematical model of solid food pasteurization by ohmic heating. The result revealed that pasteurization time is particularly in uenced by the sample electrical conductivity, since the generation term appearing in the heat equation is predominant with respect to thermal conduction. [35] studied the modi cation, fabrication and performance evaluation of ohmic heating machine. He proposed that the rate of ohmic heating depends on the electric eld distribution and the uniformity of the heating depends on the electrical conductivity and the geometry of the system. [36] studied the model for electrical conductivity of muscle meat during ohmic heating. [37] studied the concept of ohmic heating on foods. He concluded that ohmic heating is a developing innovation, with vast number of real and future applications including military eld and long duration space missions. [38] studied the emerging concept in organic synthesis using ohmic heating. [39] studied the principle, application and scope of the ohmic heating technology. [40] wrote a review on ohmic heating for food. [41] reviewed ohmic heating as an alternative preservation technique. [42] studied the changes in phytochemicals pro le of two instant corn ours produced by di erent processes. These processes were the traditional nixtamalization (TN) and the ohmic heating process. The researchers reported the highest total phenolic content in the our was recorded with ohmic heating process as compared to the TN process.

The main objective of the present paper is to investigate numerically, thermal di usion and di usion-thermo e ects on ohmic heating with MHD free convective ow past a vertical porous plate in the presence of viscous dissipation. It is observed that the present study will enhance the understanding of MHD free convective ow in heat and mass transfer in several applications. Therefore, it is of great signi cance to the researchers to study heat and mass transfer with thermal di usion and di usion thermo, ohmic heating and viscous dissipation as applied in various engineering processes.

2. Mathematical Formulation

Consider an incompressible and electrically conducting viscous uid past a vertical channel in a porous medium under the in uence of a uniform transverse magnetic eld and constant suction with viscous dissipation in the presence of Soret and Dufour e ects such that the x-axis is taken along the vertical infinite plate in upward direction and y-axis is normal to it. Since the plate is considered Infinite in the x- direction, therefore all the physical variables are function of t and y only. Let u and v be the components of velocity in x and y directions respectively, taken along and perpendicular to the plate. Under the above assumptions, the schematic diagram and general equations governing the flow is expressed as follows:

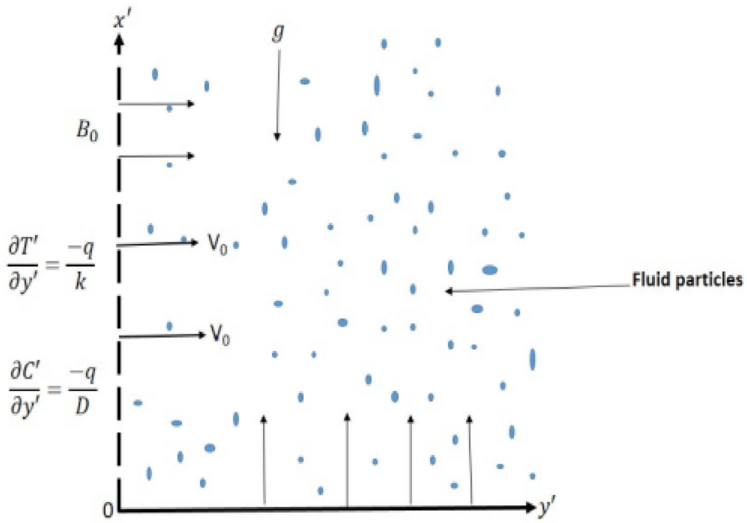

Figure 1: Schematic diagram of the problem

(2.1) $$\frac{\partial v'}{\partial y'} = 0.$$

(2.2) $$\frac{\partial u'}{\partial t'} + v_0\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - \frac{\sigma B_0^2}{\rho}u' - \frac{b}{K}u'^2 - \frac{\nu}{K}u'.$$

(2.3) $$\frac{\partial T'}{\partial t'} + v_0\frac{\partial T'}{\partial y'} = \alpha\frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 C'}{\partial y'^2} + \sigma B_0^2 u'^2.$$

(2.4) $$\frac{\partial C'}{\partial t'} + v_0\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T'}{\partial y'^2} - K C'.$$

The initial and boundary conditions are given below:

(2.5) $$\begin{aligned} &t' \le 0; \quad u' = 0; \quad T' = T'_\infty; \quad C' = C'_\infty \quad \forall y'; \\ &u' = 0; \quad \frac{\partial T'}{\partial y'} = -\frac{q}{k}; \quad \frac{\partial C'}{\partial y'} = -\frac{q_w(x)}{D} \quad \text{at} \quad y' = 0 \\ &u' = 0; \quad T' \to T'_\infty; \quad C' \to C'_\infty \quad \text{as} \quad y' \to \infty. \end{aligned}$$

where, u-velocity component, t^0 - time, g-acceleration due to gravity, - volumetric coe cient of thermal expansion, -volumetric coe cient of expansion with coe cient, T^0-temperature of the uid in the boundary layer, C^0-concentration in the boundary layer, T_1^0 -temperature of the uid far away from the plate, C_1^0 -species concentration in uid far away from the plate, -electrical conductivity, $_0$-magnetic induction, -density of the uid, -thermal di usivity, k-thermal conductivity, ν-kinematic viscosity, c_p-speci c heat at constant pressure, c_s-concentration susceptibility, T_m - mean uid temperature, k_T - thermal di usion ratio, D_m - di usion coe cient, q_r - radiation heat ux, D - species di usion coe cient, b - empirical constant, K -Darcy permeability, K -chemical reaction, q_w-mass ux per unit area, q-heat ux per unit area at the plate.
On introducing the non dimensional quantities:

(2.6)

$$U=\frac{u^0L}{vGr^{1/4}};\quad V=\frac{v^0L}{\nu Gr^{1/4}};\quad P_r=\frac{v}{};\quad S_c=\frac{v}{D};\quad Y=\frac{y^0}{LGr^{1/4}};\quad E_c=\frac{g\ L}{C_p};$$

$$t=\frac{t^0vGr^{1/2}}{L^2};\quad F=\frac{k_ek}{4\ _sT_1^{03}};\quad N=\frac{g\quad q_wL}{DGr^{5/4}v^2};\quad Gr=\frac{g\ L^4q}{kv^2};\quad C=\frac{C^0\quad C_1^0}{\frac{q_wL}{DGr^{1/4}}};$$

$$T=\frac{T^0\quad T_1^0}{\frac{qL}{kGr^{1/4}}};\quad Du=\frac{D_mK_T}{C_sC_p};\quad Sr=\frac{D_mK_T}{T_w};\quad qr=\frac{4\ _s}{3k_e}\frac{\partial T^{04}}{\partial y};\ M=\frac{{}_0^2L^2}{v};$$

$$H={}_0^2.$$

The radiative heat ux qr is given by $qr=\frac{4\ _s}{3k_e}\frac{\partial T^{04}}{\partial y}$ where $_s$ is the Stefan Boltzman constant and K_e the mean absorption coe cient, can be linearised by expanding T^{04} into the Taylor series about T_1^0, which after neglecting higher order terms takes the form, $T^{04}=4T_1^{03}T^0\quad 3T_1^{04}$. L-length of plate, Gr- Grashof number, Pr- Prandtl number, Sc-Schmidt number, F- radiation parameter, N- buoyancy ratio parameter, M- magnetic parameter, Ec- Eckert number, Sr- Soret term, Du- dufour term, H- heating term.
Applying equation(6) to equations (1)-(5) we obtain

(2.7)
$$\frac{\partial U}{\partial t}+v_0\frac{\partial U}{\partial Y}=\frac{\partial^2U}{\partial Y^2}+TGr\ \frac{1}{4}+NC\quad MU\quad BU^2\quad\frac{U}{Da}.$$

(2.8)
$$\frac{\partial T}{\partial t}+v_0\frac{\partial T}{\partial Y}=\frac{1}{Pr}\ 1+\frac{4}{3F}\ \frac{\partial^2T}{\partial Y^2}+Gr^{1/4}Ec\ \frac{\partial U}{\partial Y}^2+D_u\frac{\partial^2C}{\partial Y^2}+HU^2.$$

(2.9)
$$\frac{\partial C}{\partial t}+v_0\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2C}{\partial Y^2}+Sr\frac{\partial^2T}{\partial Y^2}\quad KrC.$$

with initial and boundary condition;

(2.10)
$$\begin{gathered} t\quad 0;\quad U=0;\quad T=0;\quad C=0\quad \forall\ Y; \\ U=0;\quad \frac{\partial T}{\partial Y}=\ 1;\quad \frac{\partial C}{\partial Y}=\ 1\quad \text{at}\quad Y=0 \\ U=0;\quad T=0;\quad C=0;\quad \text{as}\quad Y\to\infty\quad . \end{gathered}$$

In order to determine the three entities of physical interest namely, the skin friction coe cient(C_f), Nusselt number, (Nu) and Sherwood number (Sh), we use the relations;

$$C_f = \frac{\partial U}{\partial y}_{y=0}; \quad Nu = \frac{\partial T}{\partial y}_{y=0}; \quad Sh = \frac{\partial C}{\partial y}_{y=0} \tag{2.11}$$

3. Numerical Procedure

To test the validity of our results, a comparison of the present results with those of the works of [43] are performed by setting the introduced new parameters $(Da; Du; Sr; Kr; B; H)$ to zero and good agreement were obtained.
In order to solve the non-linear equations (7) to (9), under the boundary conditions (10), an implicit finite difference scheme has been employed. The finite difference equations corresponding to equations (7) to (9) are as follows;

(3.1)
$$\frac{U_i^{j+1} - U_i^j}{\Delta t} + V_0 \frac{U_{i+1}^j - U_i^j}{\Delta y} = \frac{U_{i-1}^{j+1} - 2U_i^{j+1} + U_{i+1}^{j+1}}{(\Delta y)^2} + Gr^{1/4}\left(T_i^j + N C_i^j\right) - M U_i^j - B \left(U_i^j\right)^2 - \frac{1}{Da} U_i^j :$$

$$\frac{T_i^{j+1} - T_i^j}{\Delta t} + V_0 \frac{T_{i+1}^j - T_i^j}{\Delta y} = Q \frac{T_{i-1}^{j+1} - 2T_i^{j+1} + T_{i+1}^{j+1}}{(\Delta y)^2} + W_1 \left(\frac{U_{i+1}^j - U_i^j}{\Delta y}\right)^2 + Du \frac{C_{i-1}^j - 2C_i^j + C_{i+1}^j}{(\Delta y)^2} + H \left(U_i^j\right)^2 \tag{3.2}$$

(3.3)
$$\frac{C_i^{j+1} - C_i^j}{\Delta t} + V_0 \frac{C_{i+1}^j - C_i^j}{\Delta y} = \frac{1}{Sc} \frac{C_{i-1}^{j+1} - 2C_i^{j+1} + C_{i+1}^{j+1}}{(\Delta y)^2} + Sr \frac{T_{i-1}^j - 2T_i^j + T_{i+1}^j}{(\Delta y)^2} - Kr\, C_i^j$$

The boundary condition at $Y = 0$ for temperature in the finite difference form by the 3 point formular is:

$$\frac{-3T_{i-1}^{j+1} + 4T_i^{j+1} - T_{i+1}^{j+1}}{2\Delta y} = -1 \tag{3.4}$$

The boundary condition at $Y = 0$ for concentration in the finite difference form by the 3 point formula is:

$$\frac{-3C_{i-1}^{j+1} + 4C_i^{j+1} - C_{i+1}^{j+1}}{2\Delta y} = -1 \tag{3.5}$$

The required algebraic system for momentum, temperature and concentration equations (12) to (14) subject to boundary conditions (15) and (16) is obtained as:

$$D_l U_{i\ 1}^{j+1} + D_c U_i^{j+1} + D_r U_{i+1}^{j+1} = \ 1 + r_2 \ \ M \ \ t \ \ W_3 \ \ U_i^j \ \ r_2 U_{i+1}^j \tag{3.6}$$
$$B \ \ t \ \ U_i^j \ {}^2 + W_2 T_i^j + N \ \ t C_i^j$$

where $D_l = D_r = \ \ r_1$, $D_c = 1 + 2 r_1$, $W_2 = \ \ tGr \ \ ^{1=4}$ and $W_3 = \frac{t}{Da}$

$$\frac{4}{3}B_l + B_c \ \ T_1^{j+1} + \ \ B_r \ \ \frac{B_l}{3} \ \ T_2^{j+1} = \ 1 + r_2 \ \ T_i^j \ \ r_2 T_2^j + r_3 \ \ U_2^j \ {}^2 + \tag{3.7}$$
$$r_3 + H \ \ t \ \ U_1^j \ {}^2 + \ \frac{4}{3}D_u r_1 \ \ 2D_u r_1 \ \ C_1^j + \ \ D_u \frac{r_1}{3} + D_u r_1 \ \ C_2^j$$
$$\frac{2}{3}B_l \ \ y + \frac{2}{3}D_u r_1 \ \ y$$

where $B_l = Br = \ \ Qr_1$, $B_c = 1 + 2\,Qr_1$

$$\frac{4}{3}A_l + A_c \ \ C_1^{j+1} + \ \ A_r \ \ \frac{A_l}{3} \ \ C_2^{j+1} = \ (r_2 S_c) + S_c \ \ (S_c K_r \ \ t) \ \ C_1^j \ \ r_2 S_c C_2^j + \tag{3.8}$$
$$\left(\frac{4S_cS_r r_1}{3} \ \ 2S_cS_r r_1\right)T_1^j + \left(\ \ \frac{S_cS_r r_1}{3} + S_cS_r r_1\right)T_2^j + \frac{2S_cS_r r_1}{3} \ \ y$$

where $A_l = Ar = \ \ r_1$, $A_c = Sc + 2r_1$

4. Results and Discussion

The algebraic solutions (17) to (19) for the dimensionless velocity, temperature and concentration distribution are plotted against the governing parameters in the uid ow. The velocity pro le is displayed in Figures 2-15. Figures 16-22 displays the e ect of the ow parameters on the temperature pro le while Figures 23-26 are for concentration pro le. In determining the e ects of the ow parameters in the uid ow namely, suction parameter (V_0), grashof number (G_r), buoyancy ratio parameter (N), magnetic parameter (M), forchheimer number (B), Darcy number (D_a), prandtl number (P_r), radiation parameter (F), Eckert number (E_c), schmidt number (S_c), Dufour number (D_u) and Soret number (S_r), the values $Pr = 0{:}71; Gr = Ec = Sr = Du = B = F = 0{:}1; Sc = 0{:}6; Kr = 0{:}41; Du = 0{:}1; M = 50; N = 50; Da = 0{:}01$ were used for the research unless otherwise stated. The numerical results for skin friction, Nusselt number and Sherwood number are shown in Tables (1 - 2) for various parameters of interest.

Figures 2 and 16 illustrates the e ect of Prandtl number (Pr) on the velocity and temperature pro les. It is observed that as the Prandtl number increases, the velocity pro le increases while the temperature pro le decreases. Physically, an increase in Pr means decrease in thermal conductivity of uids. An increase in the Prandtl number results in a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer and therefore heat is able to di use more rapidly away from the heated surface.

Figures 3 and 23 illustrate the e ect of schmidt number (Sc) on the velocity and concentration eld. It is observed that as the schmidt number increases, the velocity and concentration eld decreases. This is because as Sc increases, the concentration decreases and this cause the concentration buoyancy e ects to decrease yielding a reduction in the uid's velocity.

The e ect of the Soret term (Sr) on velocity and concentration pro les is shown in Figures 4 and 25. It is observed that as Sr increases, there is an increase in the velocity and concentration pro les. This is due to the fact that Sr is responsible for production of heat ux by a thermal gradient. This implies, the higher the Sr term, the higher the temperature gradient which in turn leads to higher velocity of the uids particle as well as an increased concentration of species. At $Sr = 0:1$, the concentration pro le reaches its steady state.

Figure 5 shows decrease in velocity pro le while temperature is seen to increase in Figure 17 whenever the Eckert number (Ec) is increased. This is because Ec expresses the relationship between a ows' kinetic energy and boundary layer enthalpy di erence i.e the irreversible process by means of which the work-done by a uid due to action of shear forces is transformed into heat. So for energy to be dissipated, heat is required for the work-done to be carried out hence the reason for the increase in temperature and since viscosity retards movement of uid particles, velocity is seen to decrease as shown.

Figure 6 shows the e ect of grashof number (Gr) on velocity pro le. It can be seen that as Gr increases, there is a decrease in the velocity pro le. This is because Gr signi es the relative e ects of the thermal buoyancy force to the viscous hydrodynamic force.

Figures 7 and 24 shows the e ect of chemical reaction parameter (Kr) on the velocity and concentration pro les. It is observed that as Kr increase, there is decrease in the velocity and concentration pro les. This is due to the fact that as chemical reaction proceeds, there is an increase in temperature at the boundary layer which lowers the velocity and the concentration is decreased as new products are formed in the reaction.

The e ect of radiation parameter (F) on velocity and temperature pro les can be seen in Figures 8 and 18. The velocity pro le is seen to decrease as F increases while temperature pro le is seen to increase as F increases. This is because radiation is dependent upon temperature and larger values of F provide more heat to working uid which increase the thermal condition of the uid and its thermal boundary layer.

Figures 9 and 21 show the e ect of Dufour term (Du) on velocity and temperature pro les. It is observed that as the Dufour term increases, there is an increase in the velocity and temperature pro les. This is because the Dufour term is responsible for energy ux caused by concentration gradient. Therefore, increasing Du means increased concentration gradient which in-turn leads to increased temperature as well as velocity.

Figure 10 shows the e ect of buoyancy ratio parameter (N) on velocity pro le. An increase in N leads to an increase in the velocity pro le. This is because the aiding buoyancy force on a particle enables it to move in an upward direction. Therefore increasing N increases the velocity of the uids' particle thereby allowing the hot particles at the bottom of the uid to move freely to the cold region at the top.

The e ect of the magnetic parameter (M) on the velocity pro le is shown in Figure 11. High values of M improves the contradictory force known as Lorentz force. As the magnetic parameter M increase, the Lorentz force which oppose the uid motion also increases and leads to enhanced deceleration of the ow, hence the reason for the decrease on the velocity pro le.

Figures 12, 19 and 26 show the e ect of suction parameter V_0 on the velocity, temperature and concentration pro les. It is seen that as V_0 increases, there is a subsequent increase in the three pro les. This is due to the fact that suction is a force acting on a uid caused by di erence in pressure between two regions, tending to make the uid ow from the region of higher pressure to the region of lower pressure. The force acting on the uid causes an increase in temperature which in-turn leads to increased velocity and concentration.

Figure 14 illustrates the effect of the Forchheimer term (B) on the velocity profile. It is observed that an increase in B leads to a decrease on the velocity profile. This is because the Forchheimer term represents the inertial drag, thus an increase in the Forchheimer number retards the fluid flow i.e it increases resistance to the flow and hence diminishes the heat transfer rates in the boundary layer. The Figures 15 and 22 show the effect of the ohmic heating term on the velocity and temperature profiles. There is an increase in the velocity and temperatures with an increase in the heating term. This is because ohmic heating is dependent upon the electrical conductivity and resistance of a fluid which generates heat. The higher the conductivity, the higher the temperature of the fluid which in turn leads to increased velocity.

The effects of various governing parameters on Skin friction coefficient C_f , Nusselt number Nu and Sherwood number Sh are shown in Tables 1 and 2. In order to highlight the contribution of each parameter, one parameter is varied while the rest take fixed default values. It is observed from Table 1 and 2 that an increase in any of the parameters Sc, Kr, F, M, Gr and B decrease the Skin friction while increase in any of the parameters Sr, Pr, Du, N, Da, H and V_0 causes an increase in the Skin Friction. It is also observed that an increase in any of the parameters N, F, Du and V_0 increase the Sherwood number, which improves the rate of mass transfer while increase in Pr, Sc, Kr, Sr and M leads to a decrease in the mass transfer. It is also noticed that increase the Sherwood number has no effect on the parameters Gr, Da, Ec, H and B. It is also noticed that increase in the values of Pr, Sc, Kr, Ec, Du, H and M leads to a decrease in the Nusselt number which in turn decrease the rate of heat transfer. Furthermore, increase in Ec, N, Du, Sr, F and V_0 leads to an increase in the Nusselt number. It is also seen that the Nusselt number has no effect on the Forchheimer term B; Gr and Da.

4.1. Velocity Pro les. The Velocity Pro les is as stated in the gures below:

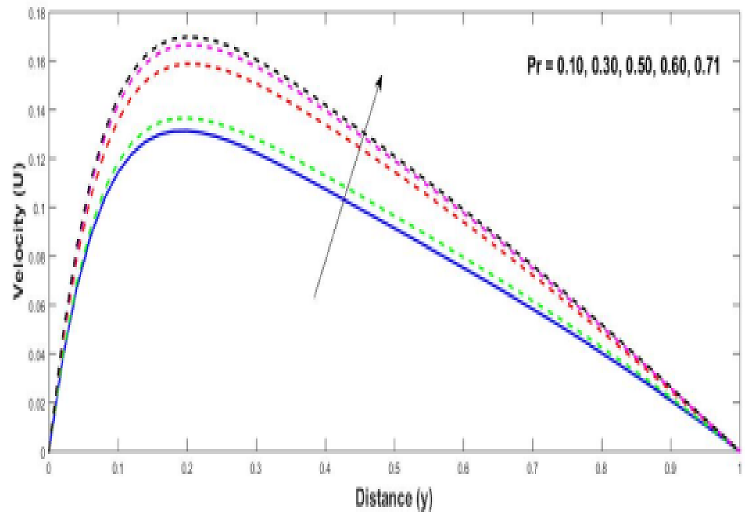

Figure 2: E ect of Prandtl Number (Pr) on Velocity Pro le when $Du = 0.1; Sr = 0.1; Gr = 0.1; Ec = 0.1; Sc = 0.6; M = 50; N = 50; Kr = 0.41; F = 0.1; Da = 0.01; V_0 = 0.1; B = 0.1$

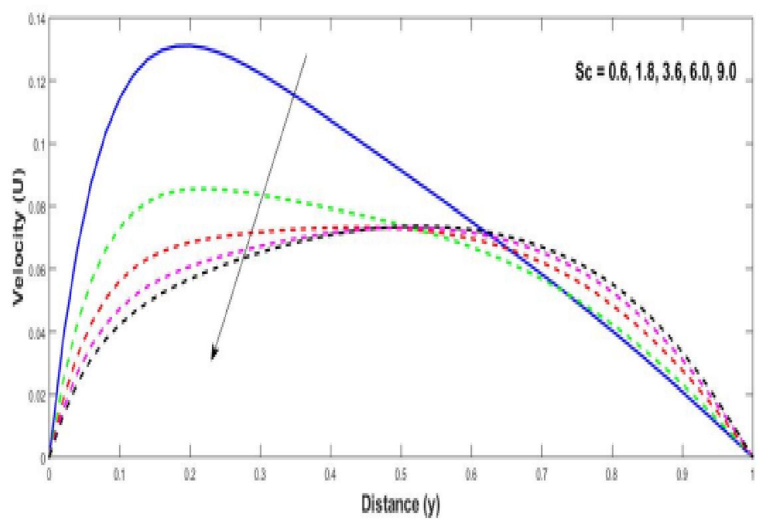

Figure 3: E ect of Schmidt Number (Sc) on Velocity Pro le when $Du = 0.1; Sr = 0.1; Gr = 0.1; Ec = 0.1; Pr = 0.71; M = 50; N = 50; Kr = 0.41; F = 0.1; Da = 0.01; V_0 = 0.1; B = 0.1$

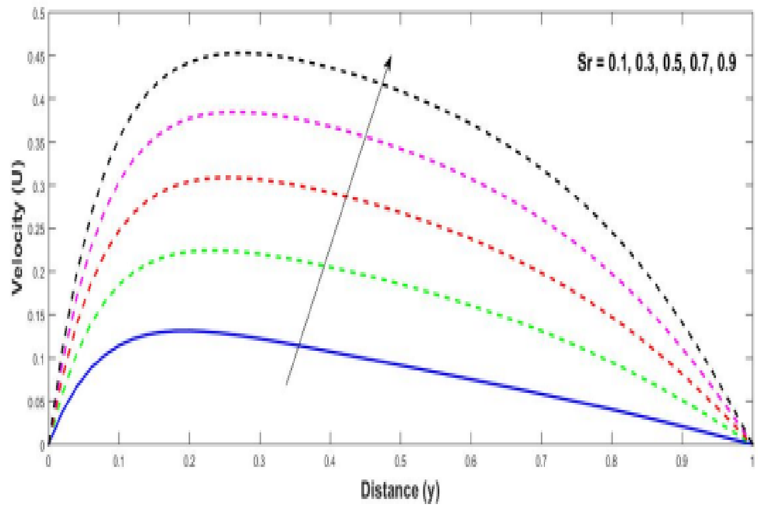

Figure 4: E ect of Soret Number (Sr) on Velocity Pro le when $Du = 0{:}1; Sc = 0{:}6; Gr = 0{:}1; Ec = 0{:}1; Pr = 0{:}71; M = 50; N = 50; Kr = 0{:}41; F = 0{:}1; Da = 0{:}01; V_0 = 0{:}1; B = 0{:}1$

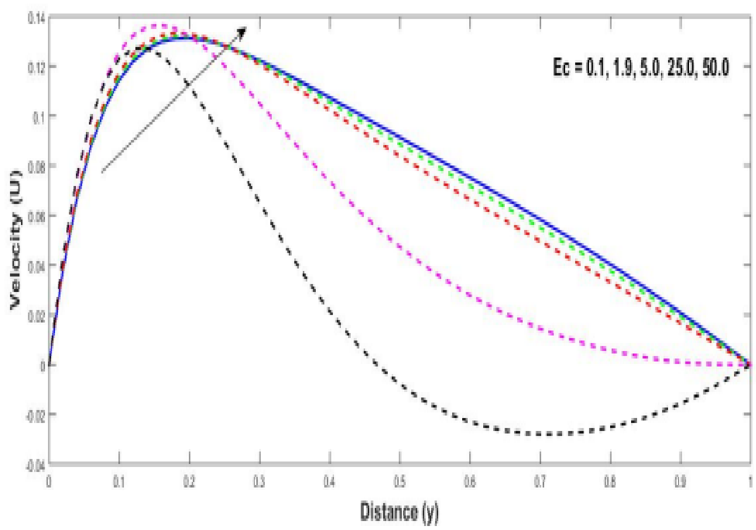

Figure 5: E ect of Eckert Number (Ec) on Velocity Pro le when $Du = 0{:}1; Sr = 0{:}1; Gr = 0{:}1; Pr = 0{:}71; M = 50; N = 50; Kr = 0{:}41; F = 0{:}1; Da = 0{:}01; V_0 = 0{:}1; B = 0{:}1$

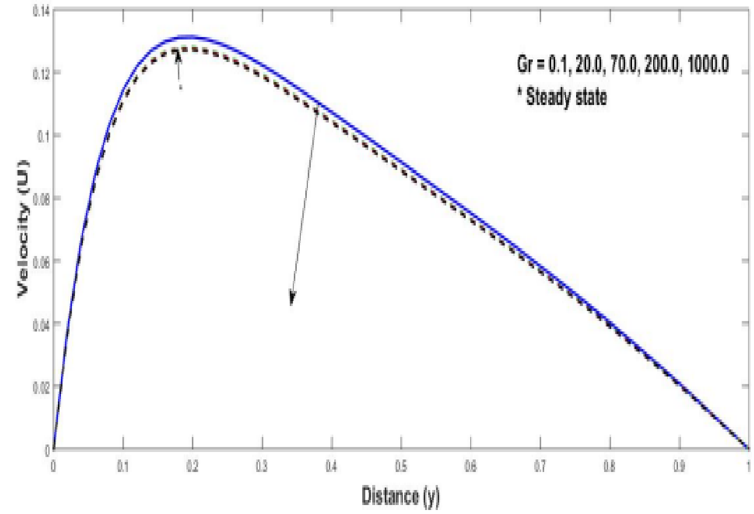

Figure 6: E ect of Grashof Number (Gr) on Velocity Pro le when $Du = 0.1; Sr = 0.1; Ec = 0.1; Pr = 0.71; M = 50; N = 50; Kr = 0.41; F = 0.1; Da = 0.01; V_0 = 0.1; B = 0.1$

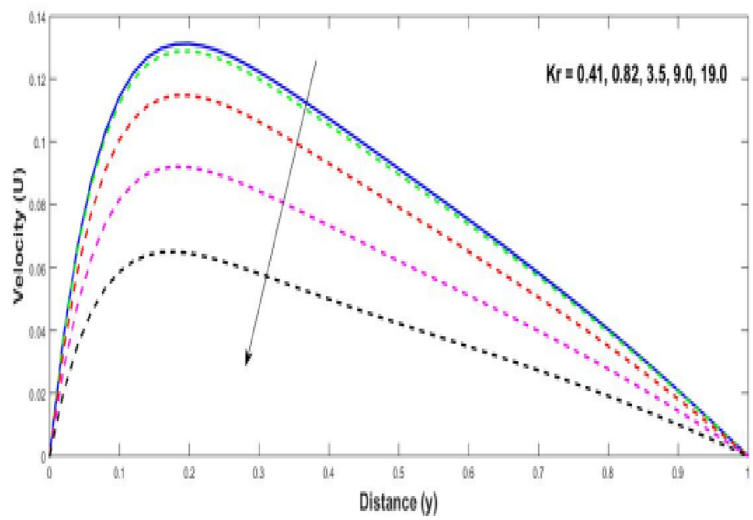

Figure 7: E ect of Chemical Reaction (Kr) on Velocity Pro le when $Du = 0.1; Sr = 0.1; Ec = 0.1; Pr = 0.71; M = 50; N = 50; Gr = 0.1; F = 0.1; Da = 0.01; V_0 = 0.1; B = 0.1$

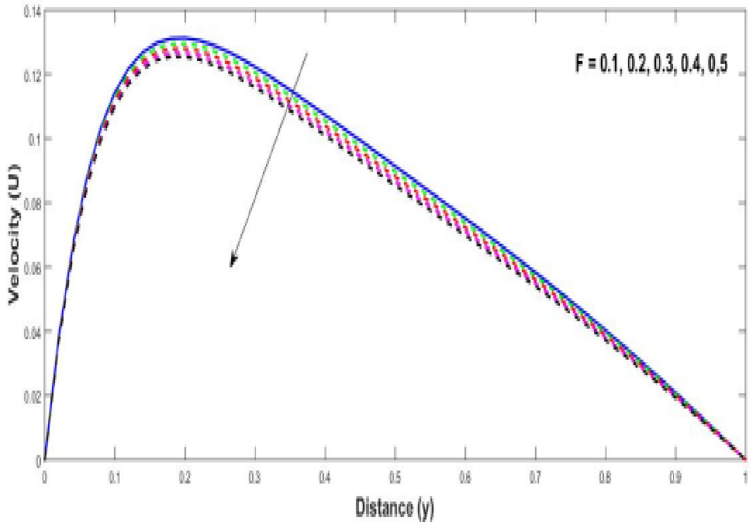

Figure 8: E ect of Radiation Parameter (F) on Velocity Pro le when $Du = 0{:}1; Sr = 0{:}1; Ec = 0{:}1; Pr = 0{:}71; M = 50; N = 50; Kr = 0{:}41; Da = 0{:}01; V_0 = 0{:}1; B = 0{:}1$

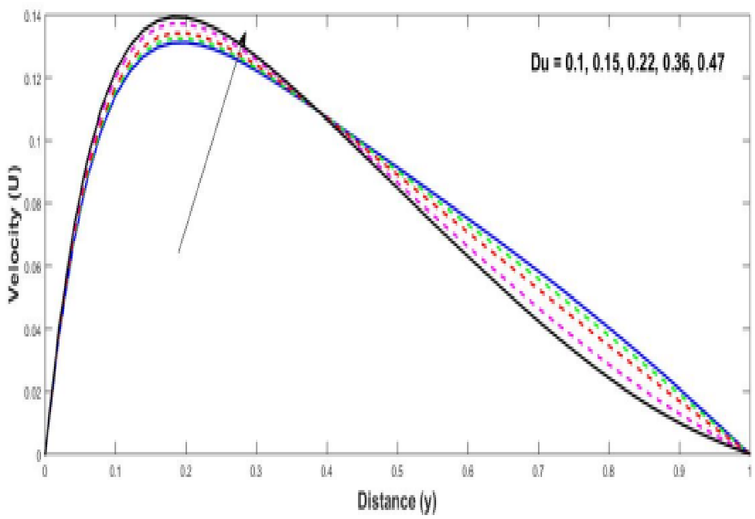

Figure 9: E ect of Dufour term (Du) on Velocity Pro le when $Sr = 0{:}1; Ec = 0{:}1; Pr = 0{:}71; M = 50; N = 50; Kr = 0{:}41; Da = 0{:}01; V_0 = 0{:}1; B = 0{:}1$

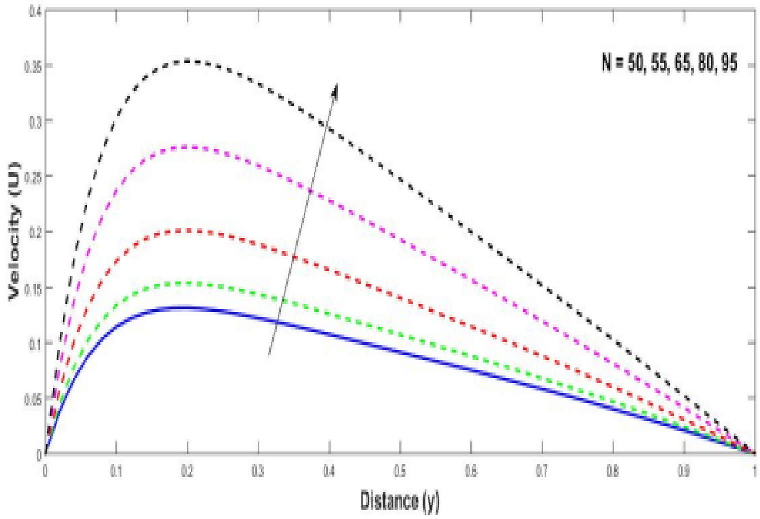

Figure 10: E ect of Buoyancy Ratio Parameter (N) on Velocity Pro le when $Du = 0{:}1; Sr = 0{:}1; Ec = 0{:}1; Pr = 0{:}71; M = 50; Kr = 0{:}41; Da = 0{:}01; V_0 = 0{:}1; B = 0{:}1$

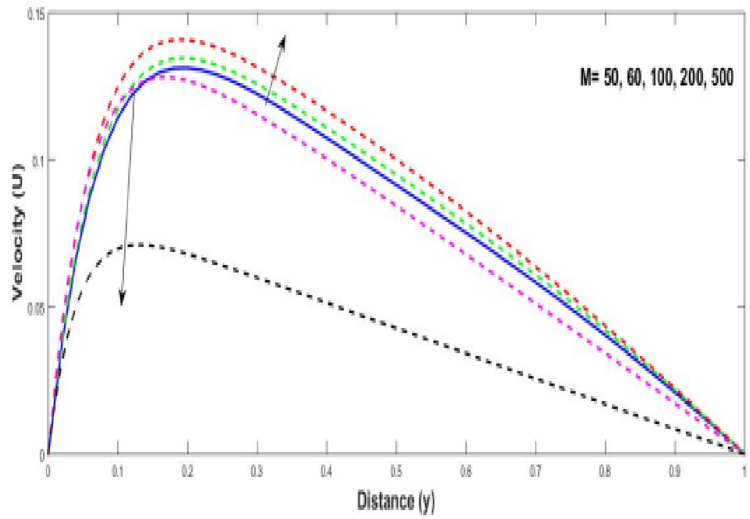

Figure 11: E ect of Magnetic Parameter (M) on Velocity Pro le when $Du = 0{:}1; Sr = 0{:}1; Ec = 0{:}1; Pr = 0{:}71; N = 50; Kr = 0{:}41; Da = 0{:}01; V_0 = 0{:}1; B = 0{:}1$

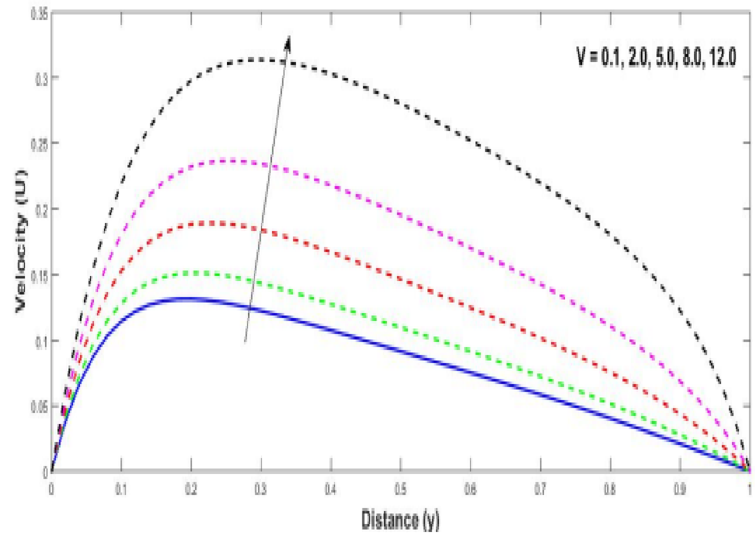

Figure 12: Effect of Suction Parameter (V_0) on Velocity Profile when
$Du = 0{:}1; Sr = 0{:}1; Ec = 0{:}1; Pr = 0{:}71; M = 50; N = 50; Kr = 0{:}41; Da = 0{:}01; B = 0{:}1$

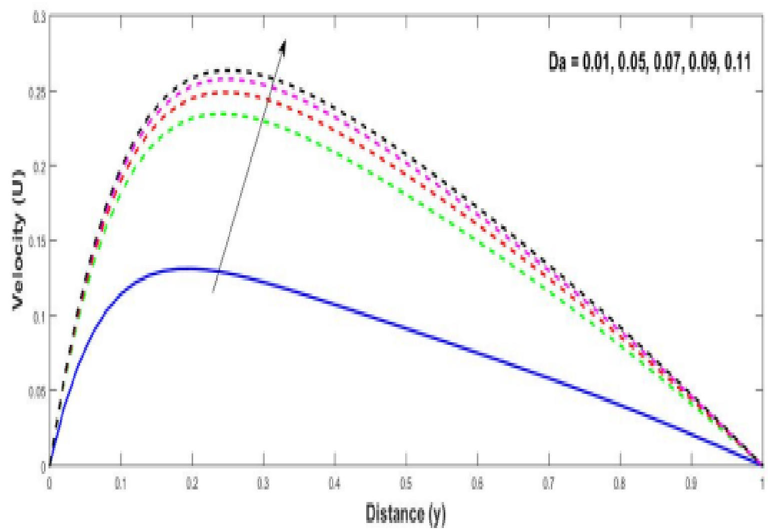

Figure 13: Effect of Darcy Number (Da) on Velocity Profile when
$Du = 0{:}1; Sr = 0{:}1; Ec = 0{:}1; Pr = 0{:}71; M = 50; N = 50; Kr = 0{:}41; V_0 = 0{:}1; B = 0{:}1$

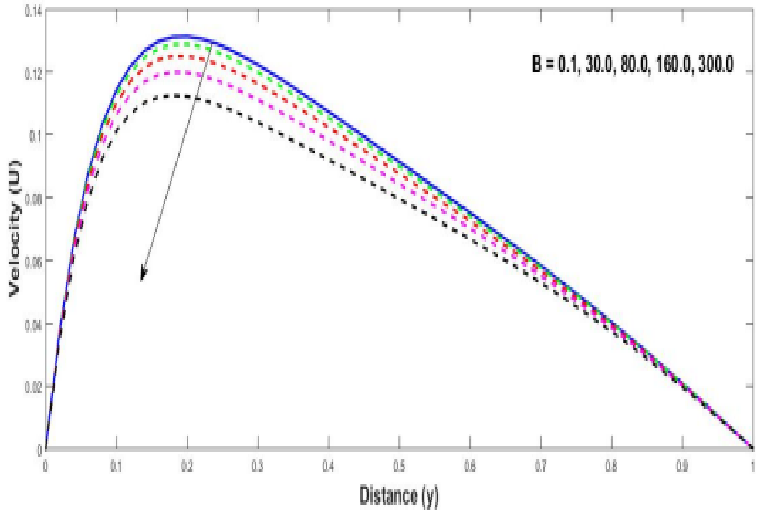

Figure 14: E ect of Forchheimer term (B) on Velocity pro le when $Du = 0:1; Sr = 0:1; Ec = 0:1; Pr = 0:71; M = 50; N = 50; Kr = 0:41; V_0 = 0:1; Da = 0:01; Gr = 0:1; F = 0:1; Sc = 0:6$

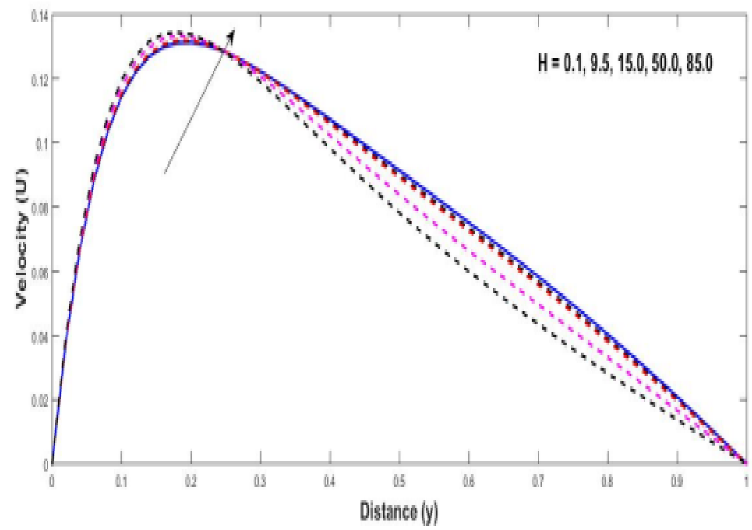

Figure 15: E ect of Ohmic Heating Parameter on Velocity Pro le when $Du = 0:1; Sr = 0:1; Ec = 0:1; M = 50; N = 50; Kr = 0:41; V_0 = 0:1; Da = 0:01; Gr = 0:1; F = 0:1; Sc = 0:6$

4.2. Temperature Profiles. The Temperature Profiles is as stated in the figures below:

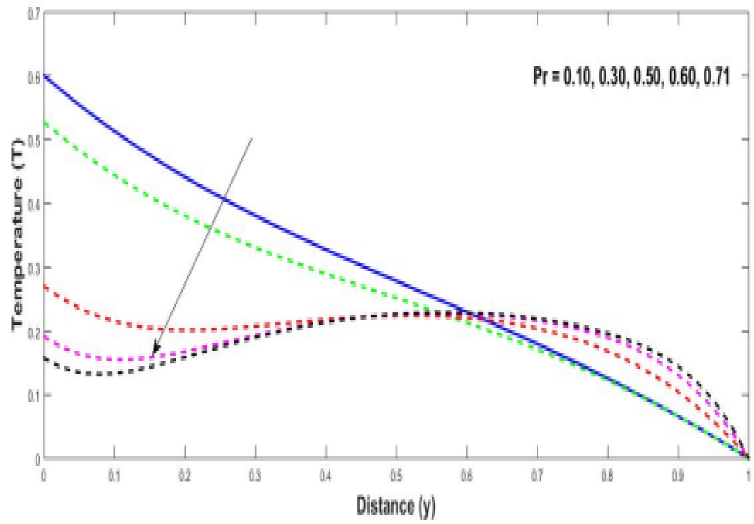

Figure 16: Effect of Prandtl Number (Pr) on Temperature Profile when $Du = 0{:}1; Sr = 0{:}1; Ec = 0{:}1; M = 50; N = 50; Kr = 0{:}41; V_0 = 0{:}1; Da = 0{:}01; Gr = 0{:}1; F = 0{:}1; Sc = 0{:}6$

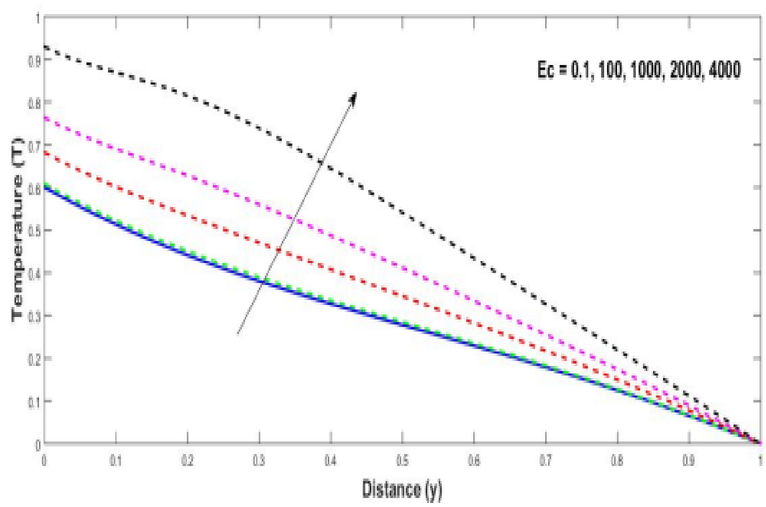

Figure 17: Effect of Eckert Number (Ec) on Temperature Profile when $Du = 0{:}1; Sr = 0{:}1; Pr = 0{:}71; M = 50; N = 50; Kr = 0{:}41; Da = 0{:}01; Gr = 0{:}1; F = 0{:}1; Sc = 0{:}6$

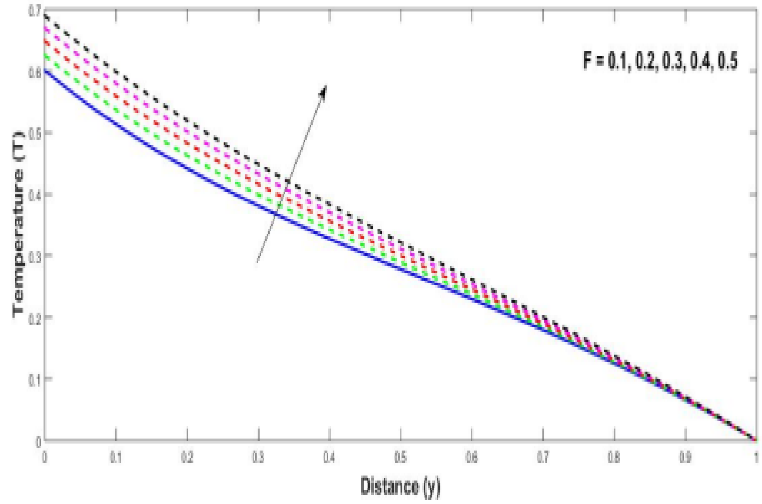

Figure 18: E ect of Radiation Parameter (F) on Temperature Pro le when $Du = 0:1; Sr = 0:1; Pr = 0:71; M = 50; N = 50; Kr = 0:41; V_0 = 0:1; Da = 0:01; Gr = 0:1; Sc = 0:6; B = 0:1$

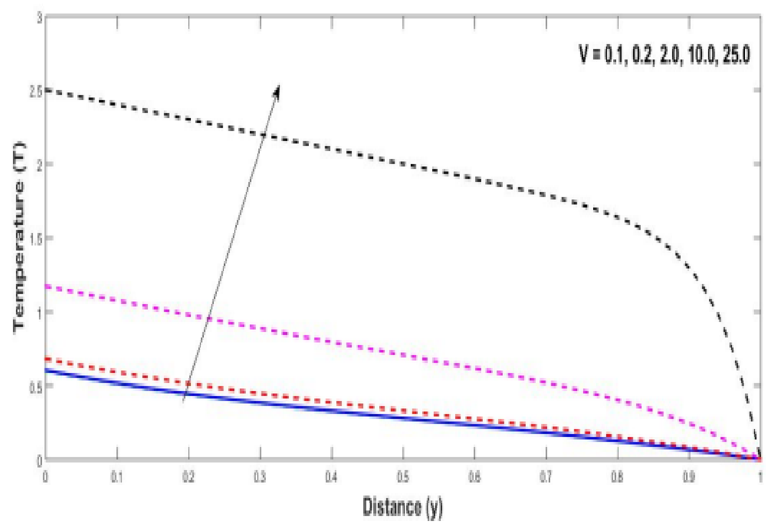

Figure 19: E ect of Suction Parameter (V_0) on Temperature Pro le when $Du = 0:1; Sr = 0:1; Pr = 0:71; M = 50; N = 50; Kr = 0:41; F = 0:1; Da = 0:01; Gr = 0:1; Sc = 0:6; B = 0:1$

Figure 20: E ect of Dufour Number (Du) on Temperature Pro le when $Gr = 0.1; Sr = 0.1; Pr = 0.71; M = 50; N = 50; Kr = 0.41; F = 0.1; Da = 0.01; V_0 = 0.1; Sc = 0.6; B = 0.1$

Figure 21: E ect of Ohmic Heating parameter (H) on Temperature Pro le when $Gr = 0.1; Sr = 0.1; Pr = 0.71; M = 50; N = 50; Kr = 0.41; F = 0.1; Da = 0.01; V_0 = 0.1; Sc = 0.6; B = 0.1$