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Steady Flow of a Third Grade Fluid in a Porous Half-Space using Adomian Decomposition Method

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Abstract

This study investigates the steady flow of a third grade fluid in a porous half space medium. The problem is a non-linear, two-point boundary value problem on semi-infinite material. The analytical solution of the problem was obtained using Adomian Decomposition Method (ADM) which is proved to be highly accurate and efficient as the series solution converges very fast. The series solution of ADM in this work is compared with the solution of other methods from literature where Shooting Method and Modified Generalized Laugure functions were previously used. ADM were proposed to provide simple way to improve the convergence of the series solution to certain number of terms. The findings demonstrated that the solution with ADM was highly accurate.

1. Introduction

During the past few decades, the flow of non-Newtonian fluids has several technical and industrial applications, especially in many real life applications like: fuel

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combustion, engineering process, polymer solutions or melts, drilling mud, hydrocarbon oils, paper, and textile industries. Out of many models which have been used to describe the non – Newtonian behavior exhibited by certain fluids, there is not a single constitutive equation available by which all the non-Newtonian fluids can be analyzed. Because of this fact, several constitutive equations for such fluids have been studied and suggested by many researchers, among them were Garg and Rajagopal (1990), Parand and Babolgham (2012).

The steady flow is the flow in which the properties at every point in the flow do not depend upon time. There is a slow change with time in the steady flow. When water flow out of a tap which has just been opened, this flow is unsteady to start with, but with time it becomes steady. Some flows, though unsteady, become steady under certain frames of reference which are referred to as pseudo steady flow (Bussuioc and Ratiu, 2003).

The fluids of the differential type have received special attention. Fluid of the second and third grade have been studied in various types of flow situations which form a subclass of the fluids of the differential type. Boundary layer theories for fluid similar to a second grade fluid have been formulated by Garg and Rajagopal (1990) developed a boundary layer approximation for a second grade fluid.

The third grade fluid is a subclass of non-Newtonian fluid which solved the system of non-linear differential equations governing the problem on the semi-infinite domain without truncating it to a finite domain. The second grade fluid model is able to predict the normal stress difference but it does not take into account the shear thinning or shear thickening phenomena that many fluids show for example, in water (Garg and Rajagopal, 1990). The third grade fluid models even for steady flow exhibits such characteristics such as honey which have higher viscosity and has porous medium, which reduces fluid flow in honey comb.

Fluids of grade three are a subclass of the family of fluids of complexity for which the constitutive law is given by the formula

(1) $T = -pI + vA + \alpha_1 A_2 + \alpha_2 A^2 + \beta_1 A_3 + \beta_2 (AA_2 + A_2 A) + \beta |A|^2 A$

where $A_1 \equiv\equiv A$, A_2 and A_3 are the first three Rivlin-Ericksen tensors (or rate-of-strain tensors) defined recursively by

$$A = A_1 = \nabla u + (\nabla u)^t \tag{2}$$

$$A_n = A_{n-1} + (\nabla u)^t A_{n-1} + A_{n-1} \nabla u \tag{3}$$

where A_n are the Rivlin- Erickson tensors defined recursively through the equation, the dot denotes the material derivative and u is the velocity field. Relation (1) arises when the fluid is assumed incompressible and the constitutive law is polynomial of degree less than 3 in the first three Rivlin-Ericksen tensors (Adriana, DragosIftimie & Marius, 2004).

In the present study, there is need to deal with the problem of non-Newtonian

fluid of third grade in a porous half space, due to the wide spread applications flow through porous media which has received substantial attention. The attempts to include porous media in the flows of the complex fluids need some new physical parameters besides non-Newtonian fluid parameters. Thus, Darcy's law for a third-grade fluid or some generation of it depending on pressure field, not neglecting porosity, are appropriate to study this type of flows thorough the porous media which is rigid or nearly rigid solid (Siddiqui, and Mohmood, 2006). Also, the modeling of polymeric flow in porous space has essential focus in specific pore-geometry models, including capillary tubes, and cylinders as described by (Hayat, Shazzad and Ayub, 2007).
Moreover, spectral method which is a class of techniques used in applied Mathematics and scientific computing to numerically solve Ordinary Differential Equation (ODE), Partial Differential Equation (PDE) has been successfully applied in the approximation of differential boundary value problems defined in unbounded domains. For problem solutions of which are sufficiently smooth, they exhibit exponential rates of convergence spectral accuracy. (Guo and Shen, 2000), Shen (2000) and Siyyam (2001) have used many approaches extensively during the last decades for the numerical solution of Partial Differential Equation (PDE) due to the accuracy when compared to Finite Differences Method (FDM) and Finite Elements Methods (FEM). The rate of convergence of spectral approximations depends only on the smoothness of the solution, yielding the ability to achieve high precision with a small number of data. Spectral collocation methods also known as pseudo spectral methods. In this project, collocation method is mainly concentrated on. In the past several years, the activity on both theory & application of spectral methods have been concentrated on collocation spectral methods one of the reasons is that collocation methods deals with non linear terms which approximate the unknown solutions in the entire computational domain by an interpolating high order polynomial at the collocation points (Wang et al, 2009).
In addition to previous approaches, Adomian Decomposition Method (ADM) has also been known to be a powerful device for solving many functional equations such as Ordinary and Partial Differential Equations, Integral Equations and so on. This ADM have been used to solve easy and more accurately a large class of system of partial differential equation with approximates that converges rapidly to accurate solution. ADM consists of splitting the equation into linear. This method of solution was introduced by the American mathematician, George Adomian (1923-1996) in search of a solution in the form of a series and on decomposing the nonlinear operator into a series in which the terms are calculated recursively using Adomian polynomials (Adomian, 1988, 1994). The main advantage of this method is that, it is capable of greatly reducing the size of computation work while still maintaining high accuracy of the numerical solution. The application of Adomian scheme has been established in a new modification of the Adomian

Decomposition Method for linear and Non-linear (Wazwaz and Al – Sayed, 2001). The ADM was first introduced by Adomian in the beginning of 1980's. The method is useful obtaining both a closed form and the explicit solution and numerical approximations of linear or nonlinear differential equations and it is also quite straightforward to write computer codes. This method has been applied to obtain a formal solution to a wide class of stochastic and deterministic problems in science and engineering involving algebraic, differential, integral-differential, differential delay, integral and partial differential equations as stated in Lesnic et al. (1999) and Dehghan (2004). The convergence of ADM for partial differential equations was presented by Cherruault(1990). Application and convergence of this method for nonlinear partial differential equations are found in (Ngarhasta, Some, Abbaoui and Cherruault, 2002) and Hashim (2006).
In general, it is necessary to construct the solution of the problems in the form of a decomposition series solution. In the simplest case, the solution can be developed as a Taylor series expansion about the function not the point at which the initial condition and integration right hand side function of the problem are determined the first term of the decomposition series for $n \geq 0$. The sum of the $u_0, u_1, u_2, \ldots$ terms are simply the decomposition series in Adomian (1989), Adomian (1994), Adomian (1998), and Dehghan (2004).

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{4}$$

Suppose that the differential equation operator including both linear and non-linear terms, can be formed as

$$Lu + Ru + Nu = F(x,t) \tag{5}$$

With initial boundary conditions as:

$$u(x,0) = g(x) \tag{6}$$

where L is the higher-order derivative which is assumed to be invertible, R is a linear differential operator of order less than L, N is the nonlinear term and *F(x,t)* is a source term. We next apply the inverse operator L^{-1} to both sides of equation (**??**) and using the given condition (5) to obtain

$$u(x,t) = g(x) + f(x,t) - L^{-1}(Ru) - L^{-1}(Nu) \tag{7}$$

where the function *f(x,t)* represents the terms arising from integrating the source term *F(x,t)* and from using the given conditions, all are assumed to be prescribed. The nonlinear term can be written as El-Sayed (2002).

$$Nu = \sum_{n=0}^{\infty} A_n, \tag{8}$$

where A_n are the Adomian polynomials. These polynomials are defined as

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[N \left(\sum_{n=0}^{\infty} \lambda^k u_k(x,t) \right) \right]_{k=0}, \; n = 0, 1, 2, \ldots . \tag{9}$$

Hence Adomian Decomposition Method (ADM) is one of the new methods for solving Initial Value Problem in Ordinary Differential Equations of various kinds arising not only in the field of Medicine, Physical and Biological Sciences but also in the area of engineering. It is important to note that a large amount of research works has been devoted to the application of (ADM) to wide class of linear and nonlinear ordinary or partial differential equations. This study, present the theoretical analysis and practical applications of (ADM). It also presents a further insight into the use of (ADM) for solving first order ordinary differential equations.

Keimanesh, Rashidi, Ali and Jafari (2011), in their paper make use of the Multi-Step Differential Transform Method (MDTM) to compute an approximate solution of the system of non-linear differential equation governing the problem. They attempted to show the reliability and performance of the MDTM in comparison with the numerical method (Fourth-order Runge-Kutta) and other analytical methods. They were able develop the first differential equation for the plane couette flow equation which serves as a useful model for many interesting problems in engineering and the likes. At the end of the research it was deduced that the multi-step differential transform method was utilized successfully to find the analytical solution of the resulting Ordinary Differential Equation. The obtained results demonstrated the reliability of the algorithm and give in a wider applicability to non linear differential equations.

Also, Yahya and Liu (2008), in their paper used modified Adomian Decomposition Method for solving initial value problem in the second order ordinary differential equations. They introduce a new reliable modification of Adomian Decomposition Method and use it (MADM) in handling a generalization of initial value problems in the second order Ordinary Differential Equation. In addition, the method is tested for some examples and the obtained results show the advantage of using this method. Their study showed that the decomposition method is simple and easy to use and produces reliable results with few iterations used. The result they obtained show that the rate of convergence of Modified Adomian Decomposition Method is higher than that of any Order Methods ever used.

Moreso, Ali and Al-Saif (2008), in their study research paper on Adomian Decomposition Method for solving some models of non linear partial differential equations said many important mathematical model can be expressed in terms of non linear partial differential equation is given by:

$$F(u,\; u_t,\; U_x,\; U_y,\; x,\; y,\; t) = 0 \tag{10}$$

with initial and boundary conditions

$$U(x,\ y,\ 0)\ =\ \phi(x,\,y),\ \forall x,\,y\in\partial,\,\Omega\in\Re^2 \tag{11}$$

$$U\ (x,y,t)\ =\ f(x,y,t)\,,\forall x,\,y\in\partial\Omega \tag{12}$$

where Ω is the solution region and $\partial\Omega$is the boundary of Ω.

Furthermore, Pue-on and Viriyapong (2012), in their paper, presented a modification Adomian Decomposition Method for solving Third-Order Ordinary Differential Equations. The purpose of their research is to present a new reliable method for solving Third Order Differential Equation and applying the method to singular and non-singular problem, a new differential operator will be established. At the end of their research, it was concluded that Modified Adomian Decomposition Method is a powerful device to solve many functional equations. In their paper, they use the method for solving a particular Third Order Ordinary Differential Equations. In some of the examples, it is decomposited that the method has the ability of applying to singular and non singular problem. The result show very outstanding rate of convergence.

Almazmumy, Hendi, Bakodah and Alzumi (2012), in their paper applied some modifications of Adomian Decomposition Method to the initial value problem in ordinary differential equation and compared the results with some previously used methods. At the end of the research, Modified Adomian Decomposition Method rate of convergence is outstanding and the result is seen to be more accurate than that of the previously used methods.

Recently, Ramana and Raghu (2014), in their paper on modified Adomian Decomposition Method for Van-der Pol equations said the Modified Adomian Decomposition Method is quite different from numerical method. They compared the results, and the modified Adomian Decomposition Method improve in accuracy and convergence, because the results are found to converge very quickly and are more accurate compared to numerical methods. They also noted in their paper that MADM is quite efficient and is practically well suited for use in solving Van-der Pol equations. They then concluded that MADM technique for dynamic solutions offer an explicit time marching algorithm that works accurately over such a bigger time step than the numerical methods like Runge Kutta Method. Different types of problems have been solved in order to confirm the robustness of the method over a wide variety of differential equations for dynamic systems like Van-der Pol non linear equations and is all the problems, they observed that MADM results very closely agree with the MATLAB® solutions.

Tatari, Mehdi and Razzaghi (2006), in their paper, investigated the application of the Modified Adomiam Decomposition Method for solving the Fokker-Planck equation and some similar equations. They applied the method to a large class of problems and less work were done compare to the traditional methods. Modified Adomian Decomposition Method decrease considerable volume for calculations,

and also they apply the Modified form of Adomiam Decomposition Method to various types of Fokker-Planck and Kelmogorous equations with given initial conditions for problems with several variables. Theoretical aspect of the methods are discussed. To present a clear overview of MADM, they select several examples with analytical solutions in one and two dimensional cases. The non linear Fokker-Planck equation is solved easily and elegantly without linearizing the problem by using MADM. At the end of their research, MADM was said to have been employed successfully for solving the Fokker-Planck equation. As it finds an exact solution of the equation using the initial condition only. It is also important to note that MADM does not require discertization of the variables, i.e. time and space, it is not affected by computation round-off errors and one is not faced with the necessity for a large computer memory and time. Decomposition approach is implemented directly in a straight forward manner without using restrictive assumptions or linearization. Comparing the results with other works. The MADM was clearly reliable if compared with mesh point technique where solution is obtained at mesh point only. It is important to note that this method unlike the most numerical techniques provide a closed form of the solution. This approach finds exact solution for non linear functional equations of various kind (Algebraic, Differential, Partial Differential, Integral) without discretizing the equation or approximating the operators and concluded that, Modified Adomian Decomposition Method avoids the difficulties and massive computational work by determining the analytic solutions.
Sennur and Guzin (2007), in the paper applied the Modified Adomian Decomposition Method to solving a non linear Sturm-Liouville problem

$$-y^{''} + y(t)^p = \lambda y\,(t) \tag{13}$$

$$y(t) \ > 0, \quad t \in I = (0,1)\,, \tag{14}$$

$$y\,(0) = y\,(1) = 0 \tag{15}$$

where p>1 is a constant and $\lambda > 0$ is an eigenvalue parameter. Also, the eigenvalus and the behaviour of Eigen functions of the problems are demonstrated.
Also recently, Yinwei and Chen (2014), in their paper on Modified Adomian Decomposition Method for double singular Boundary Value Problem (BVP) introduce a new reliable modification of Adomian Decomposition Method which gives a better approximation of the solution than the traditional one. For those problems where the standard Adomian Decomposition Method fails, Modified Adomian Decomposition Method may still converge. At the end of their research, it was concluded that Modified Adomian Decomposition Method has demonstrated that the non linear double singular BVP can be handled without difficult. The n numerical computation gives a more precise approximation of the solution. The reported result show a greater improvement over the traditional method.

Ngarhasta, Abbaoui and Cherruault (2001), in their paper on new numerical study of Modified Adomian Decomposition Method, the convergence of Modified Adomian Decomposition Method applied to linear or non linear diffusion equation. Their result show that the convergence of this method is not influenced by the choice of the linear invertible operator L in the equation.

$$L = \frac{\partial}{\partial t} \tag{16}$$

Furthermore they give some particular examples about a new canonical form where the initial value U_0 of Adomian series is chosen in some special form which verifies the initial and boundary conditions. Then Adomian Series converges to exact solution or all approximated solutions verify these conditions.
Ramesh (2010), in their paper the applied modified Adomian Decomposition Method to the solution of general Riccati differential equations. The equations under consideration include one with variable co-efficient and one with constant coefficient. The method does not need linearization, weak non linearity assumption or perturbation theory. Most of the symbolic and numerical computation were performed using Mat Lab® (Version 7.0) software. At the end of their research it was so obvious that Modified Adomian Decomposition Method has been successfully applied to find the approximate solution of the general Riccati differential equation. When compared with other tools this method gives them a very better approximation. Their new approach to linear on non-linear problem is particularly variable as a tool for scientists and Applied Mathematicians, because it provides immediate and visible symbolic terms of analytic solution as well as its numerical approximate solution to both linear and non linear problems.
Makinde (2007), they assumed that the predator in the model is not of commercial importance, they subjected the prey to constant effort harvesting with r a parameter that measures the effort being spent by a harvesting agency. The harvesting activity does not affect the predator population directly. It is obvious that the harvesting activity does reduce the predator population indirectly by reducing the availability of the prey to the predator. Adopting a simple logistic growth for prey population are $e > 0$, $b > 0$, $c > 0$ stand for the predator death rate, capturing rate and conversion rate respectively. They formulated the problems

$$\frac{\partial x}{\partial t} = x\left(1 - x\right) + \frac{bxy}{y + x} - r^{xx}, \tag{17}$$

$$\frac{\partial y}{\partial x} = \frac{e^{xy}}{y + x} - e^{y}, \tag{18}$$

where $x(t)$ and $y(t)$ represent the fractions of population densities for prey and predator at time t respectively. At the end of their research, they present some graphical results based on the decomposition method together with the numerical validation of the qualitative result obtained. Adomian Decomposition Method employed to approximate of solution of the ratio-dependent predator prey system and with constant effort prey harvesting. The semi-analytical approximations to the solutions are reliable and enhanced by Pade approximation technique and confirm the power and ability of the method as an easy device for computing the solution of a non linear system of differential equations. The method avoids the difficulties and massive computational work and usually arise from the parallel technique and finite-difference method.

Generally, the Skin friction increase with decrease radiation heat absorption but decrease with increase in the third suction at the plate. Thus, suction reduces skin friction at the plate. The rate of heat transfer at the plate decreases with an increase in the radiation heat absorption but decrease with an increase in fluid suction at the plate.

Makinde (2007), in their paper, they use Adomian Decomposition Method to a SIR epidemic model with a constant vaccination strategy and at the end of their research they were able to conclude that a Three-Compartmental deterministic mathematical model for the transmission dynamics of a childhood disease in the presence of a preventive vaccine was qualitatively and quantitatively studied. The model reveals that the disease free equilibrium is stable provided the vaccination coverage level exceeds ascertain threshold. The analytical approximations to the solutions are reliable and confirm the power and ability of the Adomian Decomposition Method as an easy device for computing the solution of a non-linear system of differential equations. The method avoids the difficulties and massive computational work that usually arise from the parallel techniques and finite-difference method.

Recently a great deal of interest has been focused on the application of Adomian's decomposition method for the solution of many different problems. For example in boundary value problems, algebraic equation and partial differential equations are considered. The adomian decomposition method, which accurately computes the series solution which is of great interest to applied science. The method, provides the solution in a rapidly convergent series with components that are elegantly computed.

The main advantage of the method is that it can be applied directly for all types of differential and integral equations, linear or non-linear, homogeneous or inhomogeneous, with constant co-efficient. Another important advantage is that the method is capable of greatly reducing the size of computation work while still maintaining high accuracy of numerical solution. Many, among 2000 mathematics sub classifications employed the Adomian technique that consists approximating the solution as infinite series (Wazwaz, 2000).

2. METHOD OF SOLUTION

In this section, we focused on Parand and Babolgham (2012) and Hayat, Shahzad and Ayub (2007); where the flow of a third grade fluid in a porous half space is unidirectional, they have generalized relation

$$(\nabla p)_x = -\frac{\mu\varphi}{k}\left(1 + \frac{\alpha_1}{\mu}\frac{\partial}{\partial t}\right)u, \tag{19}$$

for a second grade fluid to the following modified Darcy's Law for third grade fluid

$$(\nabla p)_x = -\frac{\varphi}{k}\left(\mu u + \alpha_1\frac{\partial u}{\partial t} + 2\beta_3\left(\frac{\partial u}{\partial y}\right)^2 u\right), \tag{20}$$

where μ is the dynamic viscosity, u represents the fluid velocity, p is the pressure, k and φrespectively represent the permeability and porosity of the porous half space which occupies the region $y > 0$ and α_1, β are material constants.
Here, we define the non-dimensional fluid flow velocity, f and the coordinate z as follows:

$$Z = \frac{V_0}{v}y, f(z) = \frac{u}{V_0}, V_0 = u(0) and v = \frac{\mu}{\rho} \tag{21}$$

where $u$and V_0represent the kinematic viscosity, the boundary value problem modeling the steady state flow of a third grade fluid in a porous half space according to Hayat *et al.* (2007) becomes:

$$\frac{d^2 f}{dz^2} + b_1\left(\frac{df}{dz}\right)^2\frac{d^2 f}{dz^2} - b_2 f\left(\frac{df}{dz}\right)^2 - b_3 f = 0 \tag{22}$$

subject to the following boundary conditions:

$$f(0) = 1 and f(\infty) = 0 \tag{23}$$

where the parameters $f(0) = 1$, b_2 and b_1 are defined as:

$$b_1 = 6\beta_3\frac{V_0^4}{\mu v^2}, b_2 = 2\beta_3\varphi\frac{V_0^2}{k\mu} and b_3 = \frac{\varphi v^2}{kV_0^2} \tag{24}$$

It is to be noted that the parameters are not independent, since

$$b_2 = \frac{b_1 b_3}{3} \tag{25}$$

The solution of steady flow of a third grade fluid in a porous half space is solved by using Adomian Decomposition Method (ADM). To obtain the solution of the non-linear boundary value problem equations governing the fluid flow. We rewrite equation equation (22) as follows:

$$\frac{d^2 f}{dz^2} = b_3 f + b_2 f\left(\frac{df}{dz}\right)^2 - b_1\left(\frac{df}{dz}\right)^2\frac{d^2 f}{dz^2} \tag{26}$$

Integrating equation equation (26) appropriately, we have

$$\frac{df}{dz} = a_0 + \int_0^z \left(b_3 f + b_2 f \left(\frac{df}{dz}\right)^2 - b_1 \left(\frac{df}{dz}\right)^2 \frac{d^2 f}{dz^2} \right) dz \tag{27}$$

where $a_0 = f'(0)$ is to be determined by using the other boundary conditions expressed in equation (23), we further integrate equation (27) again to obtain,

$$f(z) = 1 + \int_0^z \left(a_0 + \int_0^z \left(b_3 f + b_2 f \left(\frac{df}{dz}\right)^2 - b_1 \left(\frac{df}{dz}\right)^2 \frac{d^2 f}{dz^2} \right) dz \right) dz \tag{28}$$

More conveniently, we simplify equation (28) as follows:

$$f(z) = 1 + a_0 z + \int_0^z \left(\int_0^z \left(b_3 f + b_2 f \left(\frac{df}{dz}\right)^2 - b_1 \left(\frac{df}{dz}\right)^2 \frac{d^2 f}{dz^2} \right) dz \right) dz \tag{29}$$

The ADM requires that the approximate solution is the partial sum

$$f(z) = \sum_{n=0}^{k} f_n(z) \tag{30}$$

of the following series

$$f(z) = \sum_{n=0}^{\infty} f_n(z) \tag{31}$$

where the components $f_0, f_1, f_2, f_3, ..., f_k$are to be determined. Writing the non-linear term in (29) as a series of Adomian polynomials, we have

$$\sum_{n=0}^{\infty} A_n(z) = f_n(z) \left(\frac{df_n}{dz}\right)^2 = f_n(z) \left(f'(z)\right)^2 \tag{32}$$

and

$$\sum_{n=0}^{\infty} B_n(z) = \frac{d^2 f_n}{dz^2} \left(\frac{df_n}{dz}\right)^2 = f_n''(z) \left(f'(z)\right)^2 \tag{33}$$

such that equation (29) becomes

$$f(z) = 1 + a_0 z + \int_0^z \left(\int_0^z \left(b_3 f_n + b_2 \sum_{n=0}^{\infty} A_n(z) - b_1 \sum_{n=0}^{\infty} B_n(z) \right) dz \right) dz \tag{34}$$

and some of the Adomian polynomials for $\sum_{n=0}^{\infty} A_n(z)$ obtained from equation (34) are:

$$A_0 = f_0(\mathrm{z}) \left(f_0'(\mathrm{z})\right)^2 \ (35a) \tag{35}$$

$A_1 = f_1(\mathrm{z}) \left(f_0'(\mathrm{z})\right)^2 + 2 f_0(\mathrm{z}) f_0'(\mathrm{z}) f_1'(\mathrm{z})$ (35b)

$$A_2 = f_2(\mathrm{z})\left(f_0'(\mathrm{z})\right)^2 + 2f_1(\mathrm{z})f_0'(\mathrm{z})f_1'(\mathrm{z}) + f_0(\mathrm{z})\left(f_1'(\mathrm{z})\right)^2 + 2f_0(\mathrm{z})f_0'(\mathrm{z})f_2'(\mathrm{z}) \text{ (35c)}$$

$$\begin{aligned} A_3 &= f_3(\mathrm{z})\left(f_0'(\mathrm{z})\right)^2 + 2f_2(\mathrm{z})f_0'(\mathrm{z})f_1'(\mathrm{z}) + f_1(\mathrm{z})\left(f_1'(\mathrm{z})\right)^2 + 2f_1(\mathrm{z})f_0'(\mathrm{z})f_2'(\mathrm{z}) \\ &+2f_0(\mathrm{z})f_1'(\mathrm{z})f_2'(\mathrm{z}) + 2f_0(\mathrm{z})f_0'(\mathrm{z})f_3'(\mathrm{z}) \end{aligned} \quad (1.35\mathrm{d})$$

$$\begin{aligned} A_4 &= f_4(\mathrm{z})\left(f_0'(\mathrm{z})\right)^2 + 2f_3(\mathrm{z})f_0'(\mathrm{z})f_1'(\mathrm{z}) + f_2(\mathrm{z})\left(f_1'(\mathrm{z})\right)^2 \\ &+2f_2(\mathrm{z})f_0'(\mathrm{z})f_2'(\mathrm{z}) + 2f_1(\mathrm{z})f_1'(\mathrm{z})f_2'(\mathrm{z}) + 2f_1(\mathrm{z})f_0'(\mathrm{z})f_3'(\mathrm{z}) \quad (35\mathrm{e}) \\ &+f_0(\mathrm{z})\left(f_2'(\mathrm{z})\right)^2 + 2f_0(\mathrm{z})f_1'(\mathrm{z})f_3'(\mathrm{z}) + 2f_0(\mathrm{z})f_0'(\mathrm{z})f_4'(\mathrm{z}) \end{aligned}$$

and so on.

Also, the few Adomian polynomials for $\sum_{n=0}^{\infty} B_n(z)$ obtained from equation (34) are:

(36) $$B_0 = f_0''(\mathrm{z})\left(f_0'(\mathrm{z})\right)^2 \text{ (36}a\text{)}$$

$$B_1 = 2f_0''(\mathrm{z})f_0'(\mathrm{z})f_1'(\mathrm{z}) + \left(f_0'(\mathrm{z})\right)^2 f_1''(\mathrm{z}) \text{ (36b)}$$

$$B_2 = f_0''(\mathrm{z})\left(f_1'(\mathrm{z})\right)^2 + 2f_0''(\mathrm{z})f_0'(\mathrm{z})f_2'(\mathrm{z}) + 2f_1''(\mathrm{z})f_0'(\mathrm{z})f_1'(\mathrm{z}) + \left(f_0'(\mathrm{z})\right)^2 f_2''(\mathrm{z}) \text{ (36c)}$$

$$\begin{aligned} B_3 &= \left(f_1'(\mathrm{z})\right)^2 f_1''(\mathrm{z}) + 2f_0''(\mathrm{z})f_1'(\mathrm{z})f_2'(\mathrm{z}) + 2f_2''(\mathrm{z})f_1'(\mathrm{z})f_0'(\mathrm{z}) + 2f_0''(\mathrm{z})f_0'(\mathrm{z})f_3'(\mathrm{z}) \\ &+2f_1''(\mathrm{z})f_2'(\mathrm{z})f_0'(\mathrm{z}) + \left(f_0'(\mathrm{z})\right)^2 f_3''(\mathrm{z}) \end{aligned} \quad (36\mathrm{d})$$

$$\begin{aligned} B_4 &= f_0''(\mathrm{z})\left(f_1'(\mathrm{z})\right)^2 + 2f_0''(\mathrm{z})f_1'(\mathrm{z})f_3'(\mathrm{z}) + 2f_0''(\mathrm{z})f_0'(\mathrm{z})f_4'(\mathrm{z}) + 2f_1'(\mathrm{z})f_2'(\mathrm{z})f_1''(\mathrm{z}) \\ &+2f_0'(\mathrm{z})f_3'(\mathrm{z})f_1''(\mathrm{z}) + \left(f_1'(\mathrm{z})\right)^2 f_2''(\mathrm{z}) + 2f_0'(\mathrm{z})f_2'(\mathrm{z})f_2''(\mathrm{z}) \\ &+2f_0'(\mathrm{z})f_1'(\mathrm{z})f_3''(\mathrm{z}) + \left(f_0'(\mathrm{z})\right)^2 f_4''(\mathrm{z}) \end{aligned} \quad (36\mathrm{e})$$

and so on.

Taking the zeroth component of equation (34) and following the new modification from El-Sayed and Wazwaz (2001), we have:

(37) $$f_0(z) = 1$$

(38) $$f_1(z) = a_0 z + \int_0^z \left(\int_0^z (b_3 f_0 + b_2 A_0 - b_1 B_0)\, dz\right) dz$$

(39) $$f_{n+1}(z) = \int_0^z \left(\int_0^z (b_3 f_n + b_2 A_n - b_1 B_n)\, dz\right) dz \text{such that} n \geq 1$$

The approximate solutions are given by the partial sum in equation (30) as stated earlier on as

(40) $$f(z) = \sum_{n=0}^{k} f_n(z)$$

such that

$$f_0(z) = 1 \tag{41}$$

In order to be able to get the solution for equation (38), we differentiate equation (37) as follows:

$$f_0^{'}(\mathrm{z}) = \frac{\mathrm{df_0}}{\mathrm{dz}} = 0 \text{and} f_0^{''}(\mathrm{z}) = \frac{\mathrm{d^2f_0}}{\mathrm{dz^2}} = 0 \tag{42}$$

Using equations equation (41) and equation (42) with appropriate Adomian polynomials, in equation (35) and equation (38) we obtain the following:

$$f_1(z) = a_0 z + \int_0^z \left(\int_0^z \left(b_3 f_0 + b_2 \left(f_0(\mathrm{z}) \left(f_0^{'}(\mathrm{z}) \right)^2 \right) - b_1 \left(f_0^{''}(\mathrm{z}) \left(f_0^{'}(\mathrm{z}) \right)^2 \right) \right) dz \right) dz \tag{43}$$

Simplifying equation (43), we obtain

$$f_1(z) = a_0 z + \frac{1}{2} b_3 z^2 \tag{44}$$

The next term from equation (40) is

$$f_2(z) = \int_0^z \left(\int_0^z \left(b_3 f_1 + b_2 A_1 - b_1 B_1 \right) dz \right) dz \tag{45}$$

In a similar way, in order to get the solution of equation (44), we differentiate equation (45) as follows:

$$f_1^{'}(\mathrm{z}) = \frac{\mathrm{df_1}}{\mathrm{dz}} = a_0 + b_3 z \text{ and } f_1^{''}(\mathrm{z}) = \frac{\mathrm{d^2f_1}}{\mathrm{dz^2}} = b_3 \tag{46}$$

Using equations (41), (43) and (45) with appropriate Adomian polynomials, equations (35), (36) and (44) therefore become:

$$f_2(z) = \int_0^z \left(\int_0^z \left(\begin{array}{l} b_3 f_1 + b_2 \left(f_1(\mathrm{z}) \left(f_0^{'}(\mathrm{z}) \right)^2 + 2 f_0(\mathrm{z}) f_0^{'}(\mathrm{z}) f_1^{'}(\mathrm{z}) \right) \\ -b_1 \left(2 f_0^{''}(\mathrm{z}) f_0^{'}(\mathrm{z}) f_1^{'}(\mathrm{z}) + \left(f_0^{'}(\mathrm{z}) \right)^2 f_1^{''}(\mathrm{z}) \right) \end{array} \right) dz \right) dz \tag{47}$$

Simplifying equation (47), we conveniently obtain,

$$f_2(z) = b_3 \left(\frac{a_0 z^3}{6} + \frac{b_3 z^4}{24} \right) \tag{48}$$

We proceed to get the next term from equation (41) as:

$$f_3(z) = \int_0^z \left(\int_0^z \left(b_3 f_2 + b_2 A_2 - b_1 B_2 \right) dz \right) dz \tag{49}$$

In a similar way, in order to get the solution of (49), we differentiate (48) as follows:

$$(50)\qquad f_2^{'}(\mathrm{z}) = \frac{\mathrm{df_2}}{\mathrm{dz}} = b_3\left(\frac{a_0z^2}{2} + \frac{b_3z^3}{6}\right) \text{ and } f_2^{''}(\mathrm{z}) = \frac{\mathrm{d^2f_2}}{\mathrm{dz^2}} = b_3\left(a_0z + \frac{b_3z^2}{2}\right)$$

Using equations (42), (44), (46) and (50) with appropriate Adomian polynomials, equations (34), (35) and (49) becomes:

(51)

$$f_3(z) = \int_0^z \left(\int_0^z \left(b_3f_2 + b_2 \begin{pmatrix} f_2(\mathrm{z})\left(f_0^{'}(\mathrm{z})\right)^2 \\ +2f_1(\mathrm{z})f_0^{'}(\mathrm{z})f_1^{'}(\mathrm{z}) \\ +f_0(\mathrm{z})\left(f_1^{'}(\mathrm{z})\right)^2 \\ +2f_0(\mathrm{z})f_0^{'}(\mathrm{z})f_2^{'}(\mathrm{z}) \end{pmatrix} - b_1 \begin{pmatrix} f_0^{''}(\mathrm{z})\left(f_1^{'}(\mathrm{z})\right)^2 \\ +2f_0^{''}(\mathrm{z})f_0^{'}(\mathrm{z})f_2^{'}(\mathrm{z}) \\ +2f_1^{''}(\mathrm{z})f_0^{'}(\mathrm{z})f_1^{'}(\mathrm{z}) \\ +\left(f_0^{'}(\mathrm{z})\right)^2 f_2^{''}(\mathrm{z}) \end{pmatrix} \right) dz \right) dz$$

Simplifying equation (51) in a better form, we obtain

$$(52)\qquad f_3(z) = b_2\left(\frac{a_0^2z^2}{2} + \frac{a_0b_3z^3}{3} + \frac{b_3^2z^4}{12}\right) + b_3\left(\frac{a_0b_3z^5}{120} + \frac{b_3^2z^6}{720}\right)$$

We continue to get the next term from equation (51) as:

$$(53)\qquad f_4(z) = \int_0^z \left(\int_0^z (b_3f_3 + b_2A_3 - b_1B_3)\, dz \right) dz$$

In a similar way, in order to get the solution of equation (53), we differentiate equation (52) as follows:

$$(54)\qquad f_3^{'}(\mathrm{z}) = \frac{\mathrm{df_3}}{\mathrm{dz}} = b_2\left(a_0^2z + a_0b_3z^2 + \frac{b_3^2z^3}{3}\right) + b_3\left(\frac{a_0b_3z^4}{60} + \frac{b_3^2z^5}{120}\right) (54a)$$

and $f_3^{''}(\mathrm{z}) = \frac{\mathrm{d^2f_3}}{\mathrm{dz^2}} = b_2\left(a_0^2 + 2a_0b_3z + b_3^2z^2\right) + b_3\left(\frac{a_0b_3z^3}{15} + \frac{b_3^2z^4}{24}\right)$ (54b)

Using equations (42), (44), (46), (50) and (34) with appropriate Adomian polynomials, equations (34), (35) and (53) becomes:

(55)

$$f_4(z) = \int_0^z \left(\int_0^z \left(\begin{array}{l} b_3f_3 + b_2 \begin{pmatrix} f_3(\mathrm{z})\left(f_0^{'}(\mathrm{z})\right)^2 + 2f_2(\mathrm{z})f_0^{'}(\mathrm{z})f_1^{'}(\mathrm{z}) + f_1(\mathrm{z})\left(f_1^{'}(\mathrm{z})\right)^2 \\ +2f_1(\mathrm{z})f_0^{'}(\mathrm{z})f_2^{'}(\mathrm{z}) + 2f_0(\mathrm{z})f_1^{'}(\mathrm{z})f_2^{'}(\mathrm{z}) + 2f_0(\mathrm{z})f_0^{'}(\mathrm{z})f_3^{'}(\mathrm{z}) \end{pmatrix} \\ -b_1 \begin{pmatrix} \left(f_1^{'}(\mathrm{z})\right)^2 f_1^{''}(\mathrm{z}) + 2f_0^{''}(\mathrm{z})f_1^{'}(\mathrm{z})f_2^{'}(\mathrm{z}) + 2f_2^{''}(\mathrm{z})f_1^{'}(\mathrm{z})f_0^{'}(\mathrm{z}) \\ +2f_0^{''}(\mathrm{z})f_0^{'}(\mathrm{z})f_3^{'}(\mathrm{z}) + 2f_1^{''}(\mathrm{z})f_2^{'}(\mathrm{z})f_0^{'}(\mathrm{z}) + \left(f_0^{'}(\mathrm{z})\right)^2 f_3^{''}(\mathrm{z}) \end{pmatrix} \end{array} \right) dz \right) dz$$

Simplifying equation (51) in a better form, we obtain

$$
\begin{aligned}
f_4(z) = {} & b_2\left(\frac{a_0^3z^3}{6}+\frac{7a_0^2b_3z^4}{24}+\frac{a_0b_3^2z^5}{6}+\frac{b_3^3z^6}{36}\right)\\
& +b_3\left(\frac{a_0^2b_2z^4}{24}+\frac{a_0b_2b_3z^5}{60}+\frac{b_2b_3^2z^6}{360}+\frac{a_0b_3^2z^7}{5040}+\frac{b_3^3z^8}{40320}\right)\\
& -b_1\left(\frac{a_0^2b_3z^2}{2}+\frac{a_0b_3^2z^3}{3}+\frac{b_3^3z^4}{12}\right)
\end{aligned}
\tag{56}
$$

We continue to get the next term from equation (41) as:

$$
f_5(z) = \int_0^z\left(\int_0^z (b_3f_4+b_2A_4-b_1B_4)\,dz\right)dz \tag{57}
$$

In a similar way, in order to obtain the solution of equation (57), we differentiate equation (56) as follows:

$$
\begin{aligned}
f_4'(\mathrm{z}) & = \frac{\mathrm{df_4}}{\mathrm{dz}}\\
& = b_2\left(\frac{a_0^3z^2}{2}+\frac{7a_0^2b_3z^3}{6}+\frac{5a_0b_3^2z^4}{6}+\frac{b_3^3z^5}{6}\right)\\
& +b_3\left(\frac{a_0^2b_2z^3}{6}+\frac{a_0b_2b_3z^4}{12}+\frac{b_2b_3^2z^5}{60}+\frac{a_0b_3^2z^6}{720}+\frac{b_3^3z^7}{5040}\right)\\
& -b_1\left(a_0^2b_3z+a_0^2b_3^2z^2+\frac{b_3^3z^3}{3}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
f_4''(\mathrm{z}) & = \frac{\mathrm{d^2f_4}}{\mathrm{dz^2}}\\
& = b_2\left(a_0^2z+\frac{7a_0^2b_3z^2}{2}+\frac{10a_0b_3^2z^3}{3}+\frac{5b_3^3z^4}{6}\right)\\
& +b_3\left(\frac{a_0^2b_2z^2}{2}+\frac{a_0^2b_2b_3z^3}{3}+\frac{b_2b_3^2z^4}{12}+\frac{a_0b_3^2z^5}{120}+\frac{b_3^3z^6}{720}\right)\\
& -b_1\left(a_0^2b_3+2a_0^2b_3^2z+b_3^3z^2\right)
\end{aligned}
\tag{58}
$$

(58)

Using equations (42), (44), (46), (50), (54) and (58) with appropriate Adomian polynomials, equation (34), (35) and (57) becomes:

(59)

$$
f_5(z) = \int_0^z\left(\int_0^z\left(\begin{array}{l}
b_3f_4+b_2\left(\begin{array}{l}
f_4(\mathrm{z})\left(f_0'(\mathrm{z})\right)^2+2f_3(\mathrm{z})f_0'(\mathrm{z})f_1'(\mathrm{z})+f_2(\mathrm{z})\left(f_1'(\mathrm{z})\right)^2\\
+2f_2(\mathrm{z})f_0'(\mathrm{z})f_2'(\mathrm{z})+2f_1(\mathrm{z})f_1'(\mathrm{z})f_2'(\mathrm{z})+2f_1(\mathrm{z})f_0'(\mathrm{z})f_3'(\mathrm{z})\\
+f_0(\mathrm{z})\left(f_2'(\mathrm{z})\right)^2+2f_0(\mathrm{z})f_1'(\mathrm{z})f_3'(\mathrm{z})+2f_0(\mathrm{z})f_0'(\mathrm{z})f_4'(\mathrm{z})
\end{array}\right)\\
-b_1\left(\begin{array}{l}
f_0''(\mathrm{z})\left(f_1'(\mathrm{z})\right)^2+2f_0''(\mathrm{z})f_1'(\mathrm{z})f_3'(\mathrm{z})+2f_0''(\mathrm{z})f_0'(\mathrm{z})f_4'(\mathrm{z})\\
+2f_1'(\mathrm{z})f_2'(\mathrm{z})f_1''(\mathrm{z})+2f_0'(\mathrm{z})f_3'(\mathrm{z})f_1''(\mathrm{z})+\left(f_1'(\mathrm{z})\right)^2f_2''(\mathrm{z})\\
+2f_0'(\mathrm{z})f_2'(\mathrm{z})f_2''(\mathrm{z})+2f_0'(\mathrm{z})f_1'(\mathrm{z})f_3''(\mathrm{z})+\left(f_0'(\mathrm{z})\right)^2f_4''(\mathrm{z})
\end{array}\right)
\end{array}\right)dz\right)dz
$$

Simplifying (59) in a better form, we obtain

$$
\begin{aligned}
f_5(z) = b_2 &\left(\begin{array}{l} \frac{a_0^3 b_2 z^3}{3} + \frac{7a_0^3 b_3 z^5}{120} + \frac{a_0^2 b_2 b_3 z^4}{3} + \frac{61 a_0^2 b_3^2 z^6}{720} \\ + \frac{2a_0 b_2 b_3^2 z^5}{15} + \frac{13 a_0 b_3{}^3 z^7}{360} + \frac{b_2 b_3^3 z^6}{45} + \frac{13 b_3^4 z^8}{2880} \end{array} \right) \\
+ b_3 &\left(\begin{array}{l} \frac{a_0^3 b_2 z^5}{120} - \frac{a_0^2 b_1 b_3 z^4}{24} + \frac{a_0^2 b_2 b_3 z^6}{90} - \frac{a_0 b_1 b_3^2 z^5}{60} + \frac{11 a_0 b_2 b_3^2 z^7}{2520} \\ + \frac{a_0 b_3^3 z^9}{362880} - \frac{b_1 b_3^3 z^6}{360} + \frac{11 b_2 b_3^3 z^8}{20160} + \frac{b_3^4 z^{10}}{3628800} \end{array} \right) \\
- b_1 &\left(\frac{a_0^3 b_3 z^3}{6} + \frac{7 a_0^2 b_3^2 z^4}{24} + \frac{a_0 b_3^3 z^5}{6} + \frac{b_3^4 z^6}{36} \right)
\end{aligned} \tag{60}
$$

and the partial sum is taken to be

$$
f(z) = \sum_{n=0}^{5} f_n(z) \tag{61}
$$

Hence, we obtain

(62)

$$
\begin{aligned}
f(z) = 1 + a_0 z + b_3 &\left(\begin{array}{l} \frac{z^2}{2} + \frac{a_0 z^3}{6} + \frac{b_3 z^4}{24} + \frac{a_0^2 b_2 z^4}{24} - \frac{a_0^2 b_1 b_3 z^4}{24} + \frac{a_0 b_3 z^5}{120} + \frac{a_0 b_2 b_3 z^5}{60} \\ + \frac{a_0^3 b_2 z^5}{120} - \frac{a_0 b_1 b_3^2 z^5}{60} + \frac{a_0^2 b_2 b_3 z^6}{90} + \frac{b_3^2 z^6}{720} - \frac{b_1 b_3^3 z^6}{360} + \frac{b_2 b_3^2 z^6}{360} \\ + \frac{a_0 b_3^2 z^7}{5040} + \frac{11 a_0 b_2 b_3^2 z^7}{2520} + \frac{b_3^3 z^8}{40320} + \frac{11 b_2 b_3^3 z^8}{20160} + \frac{a_0 b_3^3 z^9}{362880} + \frac{b_3^4 z^{10}}{3628800} \end{array} \right) + \\
b_2 &\left(\begin{array}{l} \frac{a_0^2 z^2}{2} + \frac{a_0 b_3 z^3}{3} + \frac{a_0^3 z^3}{6} + \frac{a_0^3 b_2 z^3}{3} + \frac{b_3^2 z^4}{12} + \frac{7 a_0^2 b_3 z^4}{24} + \frac{a_0^2 b_2 b_3 z^4}{3} + \frac{a_0 b_3^2 z^5}{6} \\ + \frac{7 a_0^3 b_3 z^5}{120} + \frac{2 a_0 b_2 b_3^2 z^5}{15} + \frac{61 a_0^2 b_3^2 z^6}{720} + \frac{b_3^3 z^6}{36} + \frac{b_2 b_3^3 z^6}{45} + \frac{13 a_0 b_3{}^3 z^7}{360} + \frac{13 b_3^4 z^8}{2880} \end{array} \right) \\
- b_1 &\left(\frac{a_0^2 b_3 z^2}{2} + \frac{a_0 b_3^2 z^3}{3} + \frac{a_0^3 b_3 z^3}{6} + \frac{b_3^3 z^4}{12} + \frac{7 a_0^2 b_3^2 z^4}{24} + \frac{a_0 b_3^3 z^5}{6} + \frac{b_3^4 z^6}{36} \right)
\end{aligned}
$$

3. RESULTS AND DISCUSSION

In this section we present the results of our research conducted by using Adomian Decomposition Method (ADM). We compare the present method with numerical solutions previously obtained by Ahmad (2009) by using Shooting Method (SM) and Porand and Babolgham (2012) by using Modified Generalized Laguerre Functions (MGLF). We solved this problem for some typical values of parameters, $b_1 = 0.6$, $b_2 = 0.1$and $b_3 = 0.5$ as used in the previous results.

Table 1.1 Comparison between Previously obtained Solutions and ADM

Z	**SM**	**MGLF**	**ADM**
0.0	1.00000	1.00000	1.00000
0.2	0.87220	0.87261	0.87346
0.4	0.76010	0.76063	0.76198
0.6	0.66190	0.66243	0.66410
0.8	0.57600	0.57650	0.57834
1.0	0.50100	0.50144	0.50331
1.2	0.43560	0.43595	0.43785
1.6	0.32890	0.32920	0.33231
2.0	0.24820	0.24838	0.25757
$f'(0)$	$\mathbf{-0.68183}$	$\mathbf{-0.67829}$	$\mathbf{-0.\ 672594}$

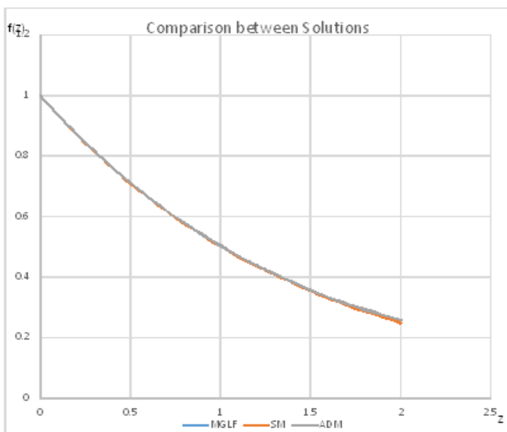

Figure 1.1

Keys:
MGLF - Modified Generalized Laguerre Functions
SM - Shooting Method
ADM - Adomian Decomposition Method

Table 1.1 shows the Comparison between Previously obtained solutions of Ahmad (2009) by using Shooting Method (SM) and Porand and Babolgham (2012) by using Modified Generalized Laguerre Functions (MGLF) and present results by using Adomian Decomposition Method (ADM). The table also indicated the numerical solution of $f'(0)$ which is important according to Porand and Babolgham (2012), in their method of solution, that is, Modified Generalized Laguerre Functions (MGLF), the value for $f'(0)$ is $\mathbf{-0.678297}$which is higher than $\mathbf{-0.681835,}$ the previous result obtained by Ahmad (2009) in using Shooting Method (SM) which according to Porand and Babolgham (2012) shows that their results is highly accurate. But, the present method of ADM gives the value of $f'(0)$ to be $\mathbf{-0.\ 672594}$ which is higher than the two previous methods. This implies that our result is more accurate than the previous results. Also, the solution is presented in Figure 1.1 as well.

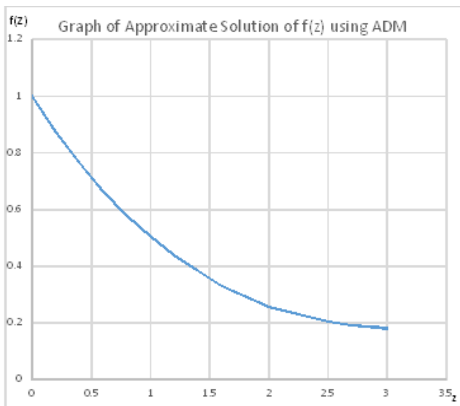

Figure 1.2 Graph of Approximate solution of $f(z)$ by ADM

Key:
ADM - Adomian Decomposition Method

Table 1.2 Comparison between Solutions using SM and ADM

Z	**SM**	**ADM**	**Absolute Error**
0.0	1.00000	1.00000	0
0.2	0.87220	0.87346	1.26×10^{-3}
0.4	0.76010	0.76198	1.88×10^{-3}
0.6	0.66190	0.66410	2.20×10^{-3}
0.8	0.57600	0.57834	2.34×10^{-3}
1.0	0.50100	0.50331	2.31×10^{-3}
1.2	0.43560	0.43785	2.25×10^{-3}
1.6	0.32890	0.33231	3.41×10^{-3}
2.0	0.24820	0.25757	9.37×10^{-3}
$f'(0)$	-0.681835	-0.672594	$\mathbf{9.241} \times 10^{-3}$

Tables 1.2 and 1.3 show the comparison of each previous methods with the present method ADM; the absolute errors were shown accordingly. Both tables show that the results are so close with few terms in the series of ADM. **Table 1.3 Comparison between Solutions using MGLF and ADM**

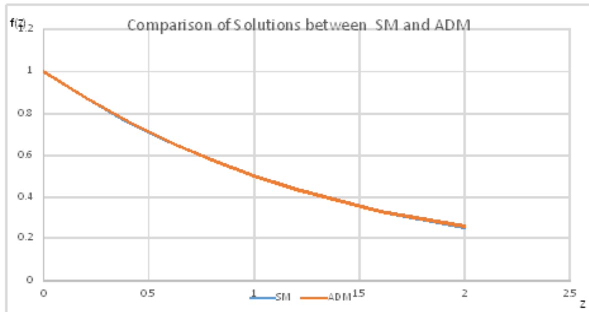

Fig 1.3

Keys:
SM - Shooting Method
ADM - Adomian Decomposition Method

Z	**MGLF**	**ADM**	**Absolute Error**
0.0	1.00000	1.00000	0
0.2	0.87261	0.87346	8.50×10^{-4}
0.4	0.76063	0.76198	1.35×10^{-3}
0.6	0.66243	0.66410	1.67×10^{-3}
0.8	0.57650	0.57834	1.84×10^{-3}
1.0	0.50144	0.50331	1.87×10^{-3}
1.2	0.43595	0.43785	1.90×10^{-3}
1.6	0.32920	0.33231	3.11×10^{-3}
2.0	0.24838	0.25757	9.19×10^{-3}
$f'(0)$	**-0.67829**	**-0. 672594**	$\mathbf{5.703} \times 10^{-3}$

The next table 1.4 shows the rapid convergence of the series solution for certain values of the flow parameters used from previously obtained solutions from other methods. It shows that at fifth term of the series solution, the result converges at that point which shows the reliability and efficiency of ADM as stated in Al-Sayed and Wazwaz; (2001). **Table 1.4 Table showing the Convergence of the solution using ADM**

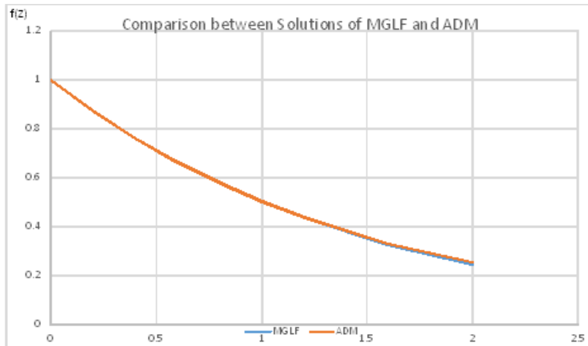

Fig 1.4

Keys:
MGLF - Modified Generalized Laguerre Functions
ADM - Adomian Decomposition Method

n	$f_n(z)$	$f(z) = \sum_{n=0}^{k} f_n(z)$
0	1.00000	1. 00000
1	0. 4226	0. 57740
2	0. 0453	0. 53210
3	0. 0123	0.54440
4	0. 0407	0. 50370
5	8.3519897×10^{-7}	0. 50370

3.1. **CONCLUSION.** Adomian Decomposition Method (ADM) was used to obtain the analytical solution of the governing equations which was used to determine the steady flow of a third grade fluid in a porous half space. The solutions obtained from the series solution proved reliable and highly accurate based on previous results.

References

[1] Adomian G. (1994): Solving frontier problems of physics: The Decomposition Method; Kluwer Academic Publishers, Dordrecht, MR1282283 (95e:00026). Zb1 0802.65122.
[2] Adomian G. (1998): A renew of the decomposition method in applied Mathematics. J. Math. Anal. Appl.; **135**: 501-544.
[3] Adomian G., Rach R. and Shawagfeh (1995): On the analytic solution of Lane-Emden equation Physics; **8** (2): 161-181.
[4] Adrjana Valentina Busuoic, Paicu M. and Iftimie D. (2004): Global existence and uniqueness of solutions for the equations of third grade fluids. Internal. J. Non-linear Mech; **39**: 1-12.

[5] Ahmad F., (2007): A simple analytic solution for the steady flow of a third grade fluid in a porous half space, Communi Nonlinear Sci. Numer. Simul.; **14**: 2848-2852.
[6] Ahmed A. and Dmitry P. (2009): Convergence of the Adomian Decomposition Method for Initial-Value Problems. Published online 3^{rd} November, 2009 in Willey Online Library. www.wileyonlinelibrary.com. DOI 10.1002/num.20549.
[7] Ali A. H. and Al-Saif A. S. J. (2008): Adomian Decomposition for Solving some models of non Linear Partial Differential Equations. Basrah Journal of Science (A); **26** (1): 1-11.
[8] Almazmumy M., Hendi F. A., Bakodah H. O. and Alzumi H. (2012): Recent Modification of Adomian Decomposition Method for Initial Value Problem in Ordinary Differential Equations. American Journal of Computational Mathematics; **2**: 228-234.
[9] Busuoic V. A. and Ratiu (2003): The second grade fluid and averaged Eular equations with Navies- slip boundary conditions. Non linearity; **16** (A203): 1119-1149.
[10] Celik E., Bayram M. and Yeloglu T. (2006): Solution of Differential-Algebraic Equations (DAEs) by Adomian Decomposition Method, International Journal of Pure and Applied Mathematical Sciences; **3** (1): 93-100.
[11] Garg V. K. and Rajagopal K. R. (1990): Stagnation point flow of a non-newtonian fluid, Mech. Res. Comm.; **17**: 415-421.
[12] Guo B. Y., Jishen and Wang Z. Q. (2000): A rational approximation and its applications to differential equations on the half line. J. Sci. Comput; **15**: 117-147.
[13] Hayat T., Shahzad F. and Ayub M. (2007): Analytical solution for the steady flow of the third grade fluid in a porous half space. Appl. Math. Model, 31: 2424-2432.
[14] Ibijola E. A., Adegboyegun B. J. and Halid O. Y. (2008): On Adomian Decomposition Method(ADM) for Numerical solution of Ordinary Equations. Journal of Advances in Natural and Applied Sciences; **2** (3): 165-169.
[15] Jazbi B., Djalalvand M., and Garshaasbi M., (2011): A numerical algorithm based on Adomian composition and product integration method to solve a class of non linear weekly singular integral equation. International Journal of Non linear Science; **11**: 353-357.
[16] Keimanesh M., Rashidi M., Ali J. and Jafari R. (2011): Study of a third grade non-Newtonian flow between two parallel plane using the multi-step differential transform method, Journal of Computer and Mathematics with Application; **62**, (8): 2871-2891.
[17] Lesnic D. and Elliot L. (1999): Modified Adomian Decomposition Method. Anal. App.; **232**: 82-98.
[18] Makinde O. D., Olajuwon B. I. and Gbolagade A. W. (2007): Adomian Decomposition approach to a boundary layer flow with terminal radiation past a moving vertical porous Plate; Downloaded 19^{th} May 2007. (www.ijamm.bc.cityu.edu.hk/).
[19] Makinde O. D. (2007): Adomian Decomposition Approach to a SIR Epidemic Model with Constant Vaccination Strategy. Journal on Applied Mathematics and Computation; **184**: 842-848. Elsevier.
[20] Makinde O. D. (2007): Solving Ratio-Dependent Predator Prey System with Constant Effort Harvesting Using Adomian Decomposition Method. Journal on Applied Mathematics and Computation; **186**: 17-22. Elsevier.
[21] Makinde O. D., Olajuwon B. I. and Gbolagade A. W. (2007): Adomian Decomposition Approach to a Boundary Layer Flow with Thermal Radiation Past a Moving vertical Porous Plate; International Journal of Applied Mathematics and Mech.; **3** (3): 62-70.
[22] Mehdi T., Mehdi D. and Mohsen R. (2007): Application of the Adomian Decomposition Method for the Fokker-Planck Equation; Journal of Mathematical and Computer Modeling; **45**: 639-650. Elsevier.

[23] Ngarhasta N., Some B., Abbaoui K. and Cherruault Y. (2002): On Numerical solution of 2-dimensional Boussinesq equations by using Homotopy Perturbation Method; kybernetes; **3**: 61-75.

[24] Parand K. and Babolgham B., (2012): Applying the modified generalized Laguerre Functions for solving steady flow of a third grade fluid in a porous half space. World Applied Science Journal; **17** (4): 467-472.

[25] Pue-on P. and Viriyapong N. (2012): Modified Adomian Decomposition Method for Solving Particular Third-Order Ordinary Differential Equations. Journal on Applied Mathematical Science; **6** (30): 1463-1469.

[26] Ramana P. V. and Raghu Prasad B. K. (2014): Modified Adomian Decomposition Method for Van der Pol Equations. International Journal of Non-Linear Mechanics; **65**, 121-132.

[27] Ramesh R. (2010): The use of Adomian Decomposition Method for solving Generalised Riccati Differential Equations: Proceedings of the 6^{th} IMT-GT Conference on Mathematics, Statistics and its application (ICMSA 2010) Universiti Tunku Abdul Rahman, Kuala Lumpur, Malaysia.; 935-941. research.utar.edu.my.

[28] Sanker E. A. (2015): Porous Medium, www.wisegeek.com/what-is-a-porous-medium.htm.

[29] Sennur S. and Guzin G. (2007): Adomian Decomposition Method for Nonlinear Sturm-Liouville Problems; Journal of Surveys in Mathematics and its application; 34L16; **2**: 11-20.

[30] Shen J. (2000): Stable and efficient spectral method in unbounded domains using Laguerre functions, SIAM Journal of Numerical Analysis, 38: 1113-1133.

[31] Siddiqui A. M., Mahmood R., and Ghori Q. K. (2006): Homotopy perturbation method for thin film flow of a third grade fluid on a moving belt. International Journal of Nonlinear Sciences and Numerical simulation;; **7**: 7-14.

[32] Siyyam, H. I. (2001): Laguerre Tau methods for solving higher order ordinary differential equations, Journal of Computation Analysis Appl.; **3**: 173-182.

[33] Tatari M., Mehdi D. and Razzaghi M. (2006): Application of Adomian Decomposition Method for the Fokker-Planck equation, Journal of Mathematical and Computer Modeling; **45**: 639-650. (ELSEVIER).

[34] Wang Z. Q., Guo B. Y. and Wu Y. N. (2009): Pseudospectral method using Generalized Laguerre functions for singular problems on Unbounded Domains; Contin. Dynamics; **11**: 1019-1038.

[35] Waseem A., Corina F., Victor T. and Constantin F. (2009): Flow of Maxwell fluid between to side walls induced by a constantly accelerating plate. Journal of Zeitschrift Fur Angewandte Mathematiik Und Physik–ZAMP.; **60** (3): 498-510.

[36] Wazwaz and El-Sayed (2001): A new modification of the Adomian Decomposition Method for linear and nonlinear operators. Journal of App. Math. Computation. **122**: 393-405.

[37] Wazwaz A. M. (2000): A new algorithm for Calculating Adomian Polynomials for non linear operation. Journal of App. Maths Computation.; **111**: 53-69.

[38] Yahya Q. H. and Liu M. Z. (2008): Modified Adomian Decomposition Method for Singular Initial Value Problem in the Second-Order Ordinary Differential Equations. Journal of Surveys in Mathematics and its Applications; **3**: 183-193.

[39] Yinwei L. and Cha'O-Kuang C. (2014): Modified Adomian Decomposition Method for Double Singular Boundary Value Problems. Rom. Journa. Phys.; **59** (5-6): 443-453.