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Some Results on Interval valued ($\in v\ q$)- fuzzy subgroups

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Abstract

In this paper, we define the concepts of interval valued ($\in\ vq$)-fuzzy subgroups and interval valued ($\in\ vq$)-fuzzy normal subgroups and discuss their properties.

1. Introduction

Zadeh [13] introduced fuzzy sets in 1965. Since then, this idea has been extended to many branches of mathematics. Fuzzy subsystem of algebraic structures have been introduced and extensively studied by several mathematicians. In particular, Rosenfeld [10] introduced the notion of fuzzy subgroups in 1971. Liu [8] defined and characterized fuzzy normality of a fuzzy subgroup in 1982. Ajmal and Thomas [1] introduced the concepts of fuzzy quasinormality. Fuzzy subrings, fuzzy ideals in a ring and the likes have also been worked upon by several researchers.

Ming and Ming [9] defined fuzzy singleton and used the idea to introduce and characterize the concepts of quasi coincidence of a fuzzy point with a fuzzy set. Bhakat and Das [2 − 5] used these concepts by Ming and Ming to introduce and characterize the concepts of ($\in vq$)- fuzzy subgroups, ($\in vq$)-fuzzy normal subgroups, ($\in vq$)-fuzzy quasi-normal subgroup, and so on.

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Zadeh [14], in order to take care of imprecision that could arise as a result of using fuzzy sets, introduced the idea of interval valued fuzzy sets. This idea has been widely used by a lot of researchers. Dong and Chul [7], introduced and characterized interval valued $(\in, \in vq)$-fuzzy ideal in a ring, Sargunadevi, *et.al.* [11, 12] studied anti interval valued fuzzy subbigroup and interval valued fuzzy subbigroup.
In this paper, we define and characterize the interval valued $(\in, \in vq)$-fuzzy subgroups as an extension of the notion of $(\in, \in vq)$-fuzzy subgroups. In particular, we define the $(\in, \in vq)$-fuzzy subgroups, $(\in, \in vq)$-normal fuzzy subgroups, and give the conditions for an interval valued fuzzy subset A of a group G to be an interval valued $(\in, \in vq)$-fuzzy subgroups, $(\in, \in vq)$-fuzzy normal subgroups of G among others.

2. Preliminaries

We present here some basic concepts and clarify notions which are used in the sequel. For details, please refer to [1-14].
Let G be a non empty set.

Definition (2.1) [Zadeh] [13 A mapping $\mu : G \to [0, 1]$ is called a fuzzy subset of G.

Definition (2.2) Let μ be a fuzzy set in a set G. Then, the level subset μ_t is defined as:
$\mu_t = \{x \in G : \mu(x) \geq t\} \ for \ t \in [0, 1]$.

Definition (2.3) [Rosenfeld] [10] Let μ be a fuzzy set in a group G. Then, μ is said to be a fuzzy subgroup of G, if the following hold:
(i) $\mu(xy) \geq \min\{\mu(x), \mu(y)\} \ \forall \ x, y \in G$;
(ii) $\mu(x^{-1}) = \mu(x) \ \forall \ x \in G$.

Definition (2.4) Let μ be a fuzzy subgroup of G. Then, the level subset μ_t, for $t \in Im\mu$ is a subgroup of G and is called the level subgroup of G.

Definition (2.5) [Ming and Ming] [9] A fuzzy subset μ of a group G of the form:

$$\mu(y) = \begin{cases} t(\neq 0) & \text{if} \quad y = x \\ 0 & \text{if} \quad y \neq x \end{cases}$$

is said to be a fuzzy point with support x and value t and is denoted by x_t

Definition (2.6) [Ming and Ming] [9] A fuzzy point x_t is said to belong to(resp. be quasi-coincident with) a fuzzy set μ , written as $x_t \in \mu$ (resp.$x_t \ q \ \mu$)

if $\mu(x) \geq t$(resp. $\mu(x) + t > 1$).

"$x_t\ \in \mu\ or\ x_t\ q\ \mu$" will be denoted by $x_t\ \in vq\ \mu$)

Definition (2.7) [Bhakat and Das] [5] A fuzzy subset μ of G is said to be an ($\in vq$)- fuzzy subgroup of G if for every $x, y\ \in G$ and $t, r\ \in (0, 1]$:
(i) $x_t\ \in \mu, y_r\ \in \mu \Rightarrow (xy)_{M(t,r)}\ \in vq\ \mu$
(ii) $x_t\ \in \mu \Rightarrow (x^{-1})_t\ \in vq\ \mu$.

Theorem (2.8) [Bhakat and Das] [5] (i) A necessary and sufficient condition for a fuzzy subset μ of a group G to be an $(\in, \in vq)$-fuzzy subgroup of G is that $\mu(xy^{-1}) \geq M(\mu(x), \mu(y), 0.5)$ for every $x,\ y\ \in G$.

(ii). Let μ be a fuzzy subgroup of G. Then $\mu_t = \{x\ \in G : \mu(x) \geq t\}$ is a fuzzy subgroup of G for every $0 \leq t \leq 0.5$. Conversely, if μ is a fuzzy subset of G such that μ_t is a subgroup of G for every $t \in (0, 0.5]$, then μ is an fuzzy $(\in, \in vq)$-fuzzy subgroup of G

Definition (2.9) [Bhakat and Das][5] Let X be a non empty set. The subset $\mu_t = \{x\ \in X : \mu(x) \geq t\}$ or $\{\mu(x) + t > 1\}$=$\{x\ \in\ X : x_t\ \in vq\ \mu\}$ is called ($\in vq$)-level subset of X determined by μ and t.

Theorem (2.10) [Bhakat and Das] [5] A fuzzy subset μ of G is a fuzzy subgroup of G if and only if μ_t is a subgroup for all $t \in (0, 1]$

Definition (2.11) [Bhakat and Das][6] An $(\in, \in vq)$-fuzzy subgroup μ of $G$is said to be an $(\in, \in vq)$-fuzzy normal subgroup of G if for any $x,\ y,\ \in G$ and $t\ \in (0, 1]$, $x_t\ \in\ \mu \Rightarrow (yxy^{-1})_t\ \in vq\ \mu$

Remark (2.12) [Bhakat and Das] [6] The following statements are equivalent:
(i) μ is $(\in, \in vq)$-fuzzy normal
(ii) $\mu(y^{-1}xy)\ \geq\ M(\mu(x), 0.5)\ \forall\ x, y\ G$
(iii) $\mu(xy)\ \geq\ M(\mu(yx), 0.5)\ \forall\ x, y\ G$.

Theorem (2.13) [Bhakat and Das] [6] Let μ be an $(\in, \in vq)$-fuzzy normal subgroup of G. Then $\{x\ \in G : \mu(x) \geq t\}$ is a normal subgroup of G for every $0 \leq t \leq 0.5$. Conversely, if μ is a fuzzy subset of G such that μ_t is a normal subgroup of G for every $0 \leq t \leq 0.5$, then μ is $(\in, \in vq)$-fuzzy normal subgroup of G.

We now give some information extracted from [7] on the utilized notion interval value sets that we also found useful for our work here.
By an interval number $\hat{a}$, we mean an interval $[\underline{a}, \bar{a}]$, where $0 \leq \underline{a} \leq \bar{a} \leq 1$. Let $D[0,1]$ denotes the set of all interval numbers. Consider the interval numbers $\hat{a}\ , \hat{b}\ \in D[0,1]$, we define refined minimum, refined maximum, refined infimum, and refined suprimum of $\hat{a}\ , \hat{b}\ \in D[0,1]$ denoted respectively by $r\min(\hat{a},\hat{b})$, $r\max(\hat{a},\hat{b})$, $r\inf(\hat{a},\hat{b})$, $r\sup(\hat{a},\hat{b})$ as follows:

$$\begin{aligned} r\min(\hat{a},\hat{b}) &= [\min\{\underline{a},\underline{b}\}, \min\{\bar{a},\bar{b}\}] \\ r\max(\hat{a},\hat{b}) &= [\max\{\underline{a},\underline{b}\}, \max\{\bar{a},\bar{b}\}] \\ r\inf(\hat{a},\hat{b}) &= [\wedge\{\underline{a},\underline{b}\}, \wedge\{\bar{a},\bar{b}\}] \\ r\sup(\hat{a},\hat{b}) &= [\vee\{\underline{a},\underline{b}\}, \vee\{\bar{a},\bar{b}\}] \end{aligned}$$

We also defined the symbols $" \leq "," = "," < "$, in case of any two interval numbers in $D[0,1]$.
(i) $\hat{a} \leq \hat{b}$ if and only if $\underline{a} \leq \underline{b}$ and $\bar{a} \leq \bar{b}$
(ii) $\hat{a} = \hat{b}$ if and only if $\underline{a} = \underline{b}$ and $\bar{a} = \bar{b}$
(iii) $\hat{a} < \hat{b}$ if and only if $\underline{a} < \underline{b}$ and $\bar{a} < \bar{b}$
Under these notations, the concept of interval -valued fuzzy sets defined on a non-empty set X as objects have the form
$A = \{(x, [\underline{\mu}_A(x), \bar{\mu}_A(x)])\}$, for every $x \in X$.
This will be briefly denoted by $A = [\underline{\mu}_A, \bar{\mu}_A]$, where $\underline{\mu}_A$ and $\bar{\mu}_A$ are two fuzzy sets in X such that $\underline{\mu}_A(x) \leq \bar{\mu}_A(x)$ for all $x \in X$
Let $\hat{\mu}_A(x) = [\underline{\mu}_A, \bar{\mu}_A]$ for every $x \in X$. Then $\hat{\mu}_A(x) \in D[0,1]$,for every $x \in X$, and therefore, the interval valued fuzzy set A is given by $A = \{x, \hat{\mu_A}(x)\}$, for every $x \in X$, where $\hat{\mu_A} : X \to D[0,1]$. For a given interval valued fuzzy sets A and B in a set X, we define
$A \subseteq B \iff for\ every\ x \in X,\ \mu_A(x) \leq \mu_B(x))$.
$A = B \iff A \subseteq B\ and\ B \subseteq A$.
$A \cap B = \{x,\ [\min\{\underline{\mu}_A(x), \underline{\mu}_B(x)\}, \min\{\bar{\mu}_A(x), \bar{\mu}_B(x)\}] \mid x \in X\}$
$A \cup B = \{x,\ [\max\{\underline{\mu}_A(x), \underline{\mu}_B(x)\}, \max\{\bar{\mu}_A(x), \bar{\mu}_B(x)\}] \mid x \in X\}$.

3. Interval valued $(\in vq)$-fuzzy subgroups

In what follows, G is a group.
Definition (3.1) An interval valued fuzzy set A in G of the form:

$$\hat{\mu}_A(y) = \begin{cases} \hat{a} \neq [0,0] & \text{if} \quad y = x, \\ [0,0] & \text{if} \quad y \neq x \end{cases}$$

is called a fuzzy interval value with support x and interval value $\hat{a}$ and is denoted by $U(x, \hat{a})$, just as in [3]. Throughout this paper, we assume that:
(i) Any two interval numbers of $D[0, 1]$ are comparable
(ii) $r\min[\hat{\mu}_A(x), \hat{\mu}_A(x)] < [0.5, 0.5]$ or $\min[\hat{\mu}_A(x), \hat{\mu}_A(x)] \geq [0.5, 0.5]$.

Definition (3.2) A fuzzy interval value $U(x, \hat{a})$ is said to belong to (resp. quasi-coincident with) an interval valued fuzzy set A, written as $U(x, \hat{a}) \in A$ (resp.$U(x, \hat{a})\ q\ A$) if $\hat{\mu}_A(x) \geq \hat{a}$ (resp. $\hat{\mu}_A(x) + \hat{a}) > [1, 1]$)
$U(x, \hat{a}) \in A$ or $U(x, \hat{a})\ q\ A$ will be denoted by $U(x, \hat{a}) \in vq\ A$ while $U(x, \hat{a}) \bar{\in} A$, $U(x, \hat{a}) \in \bar{v}qA$ will mean $U(x, \hat{a}) \in A$, $U(x, \hat{a}) \in vq\ A$ do not hold.

Definition (3.3) An interval valued fuzzy set A in G is called a $(\in, \in vq)$-fuzzy subgroup of G if for all $x, y \in G$ and $\hat{a}, \hat{b} \in D[0, 1]$
(i) $U(x, \hat{a}) \in A, U(y, \hat{b}) \in A \Longrightarrow U(xy, r\min[\hat{a}, \hat{b}]) \in vq\ A$
(ii) $U(x, \hat{a}) \in A \Longrightarrow U(x^{-1}, \hat{a}) \in vq\ A$

Theorem (3.4) A necessary and sufficient condition for an interval valued fuzzy subset A of G to be an $(\in, \in v\ q)$-fuzzy subgroup of G is that

$$\hat{\mu}_A(xy^{-1}) \geq r\min[\hat{\mu}_A(x), \hat{\mu}_A(y), [0.5, 0.5]]$$

for all $x, y \in G$.
Proof Since the condition of the theorem, according to remark(3.2) of [2], is equivalent to

$$\hat{\mu}_A(xy) \geq r\min[\hat{\mu}_A(x), \hat{\mu}_A(y), [0.5, 0.5]]\ (1^*)$$

$$\hat{\mu}_A(x^{-1}) \geq r\min[\hat{\mu}_A(x), [0.5, 0.5]]\ (2^*)$$

we shall use the approaches of [1] and [2] to establish the proof.
First, we show that $1 \Longrightarrow (1^*)$
Assume that 1 holds but (1^*) is not true, then there exists $x, y \in G$ such that

$$\hat{\mu}_A(xy) < r\min[\hat{\mu}_A(x), \hat{\mu}_A(y), [0.5, 0.5]]$$

Under this situation, the two possible cases are

$$r\min[\hat{\mu}_A(x), \hat{\mu}_A(y)] < [0.5, 0.5]$$

$$r\min[\hat{\mu}_A(x), \hat{\mu}_A(y)] \geq [0.5, 0.5]$$

For the first case, we have

$$\hat{\mu}_A(xy) < r\min[\hat{\mu}_A(x), \hat{\mu}_A(y)].$$

If $\hat{a}$ is chosen such that

$$\hat{\mu}_A(xy) < \hat{a} < r\min[\hat{\mu}_A(x), \hat{\mu}_A(y)],$$

then $U(x, \hat{a}) \in A$,and $U(y, \hat{a}) \in A$. Since $U(xy, \hat{a}) \bar{\in} A$, and $U(xy, \hat{a})\ \bar{q}A$, imply that $U(xy, \hat{a}) \in\bar{\ } vq\ A$, which is a contradiction to 1.
For the second case, we have

$$\hat{\mu}_A(xy) < [0.5, 0.5] \leq r \min[\hat{\mu}_A(x), \hat{\mu}_A(y)]$$

.
This implies that $U(x, [0.5, 0.5]) \in A$ and $U(y, [0.5, 0.5]) \in A$,
but $U(xy, [0.5, 0.5]) \in\bar{\ } vqA$, which is a contradiction to (1^*). Therefore, if 1 holds then (1^*) is true.
We now show that $((1^*)) \Longrightarrow 1$
Let $x, y \in A$ and $\hat{a}, \hat{b} \in (D[0, 1]$ be such that $U(x, \hat{a}) \in A$,and $U(y, \hat{b}) \in A$, then $\hat{\mu}_A(x) \geq \hat{a}$ and $\hat{\mu}_A(y) \geq \hat{b}$. From (1^*),

$$\hat{\mu}_A(xy) \geq r \min[\hat{\mu}_A(x), \hat{\mu}_A(y), [0.5, 0.5]] \geq r \min[\hat{a}, \hat{b}, [0.5, 0.5]].$$

If $r \min[\hat{a}, \hat{b}] < [0.5, 0.5]$, then $\hat{\mu}_A(xy) \geq r \min[\hat{a}, \hat{b}]$.
If $r \min[\hat{a}, \hat{b}] \geq [0.5, 0.5]$, then $\hat{\mu}_A(xy) \geq [0.5, 0.5]$.
These then imply that

$$\hat{\mu}_A(xy) + r \min[\hat{a}, \hat{b}] \geq [1, 1]$$

Hence,

$$U(xy, r \min[\hat{a}, \hat{b}]\ \in vq\ A)$$

which shows that 1 holds.
We also show that $2 \Longrightarrow (2^*)$. To this end, Assume that (2^*) is not valid, then there exists $x \in G$ such that

$$\hat{\mu}_A(x^{-1}) < r \min[\hat{\mu}_A(x), [0.5, 0.5]],$$

we look at it for the two cases

$$\hat{\mu}_A(x) < [0.5, 0.5],\ and\ \hat{\mu}_A(x) \geq [0.5, 0.5],$$

For the first case, we have

$$\hat{\mu}_A(x^{-1}) = \hat{r} < \mu_A(x) = \hat{t}.$$

If $\hat{a}$ is chosen such that

$$\hat{r} < \hat{a} < \hat{t}\ and\ \hat{r} + \hat{t} < [1, 1],$$

then we have that $U(x : [0.5, 0.5])\ \in\ A$. But $U(x^{-1}, [0.5, 0.5]) \in\bar{\ } vq\ A$, which is a contradiction to (2^*).
For the second case, we have $\hat{\mu}_A(x^{-1}) < [0.5, 0.5]$, which implies that $U(x^{-1}, [0.5, 0.5]) \in\bar{\ } vq\ A$. But $U(x, [0.5, 0.5]) \in A$, which is a contradiction to (2^*). Therefore, (2^*) holds.
We conclude the proof by showing that (2^*) implies 2.

Let $x \in G$ and $\hat{a} \in D[0,1]$ be such that $U(x,\hat{a}) \in A$, then $\hat{\mu_A}(x) \geq \hat{a}$. Now we have

$$\hat{\mu}_A(x^{-1}) \geq r\min[\hat{\mu}_A(x),[0.5,0.5]] \geq r\min[\hat{a},[0.5,0.5]]$$

If $\hat{a} \leq [0.5,0.5]$, then we have $\mu_A(x^{-1}) \geq \hat{a}$, and
if $\hat{a} \geq [0.5,0.5]$, then we have $\mu_A(x^{-1}) \geq \hat{a}$. Hence,
$U(x^{-1},\hat{a}) \in vq\ A$. Therefore ($2^*$) holds, and that concludes the proof.

Definition (3.5) Let $\hat{t} \in D[0,1]$. Let A be an interval valued fuzzy subset of G. The fuzzy interval level subset $\hat{A}_{\hat{t}}$ of G is defined as

$$\hat{A}_{\hat{t}} = \{x \in G : \hat{\mu}_A(x) \geq \hat{t}\}$$

Theorem (3.6) An interval valued fuzzy subsets A of a group G is an interval valued $(\in, \in vq)$ fuzzy subgroup of G if and only if $\hat{A}_{\hat{t}}$ is a subgroup of G for all $\hat{t} \in D[0,1]$

Proof suppose A is an interval valued fuzzy subgroup of G. Let $x,y \in A$ such that $x,y \in \hat{A}_{\hat{t}}$, then

$$\hat{\mu}_A(x) \geq \hat{t}\ or\ \hat{\mu}_A(x) + \hat{t} > [1,1]$$

and

$$\hat{\mu}_A(y) \geq \hat{t}\ or\ \hat{\mu}_A(y) + \hat{t} > [1,1]$$

so that

$$\hat{\mu}_A(xy^{-1}) \geq r\min[\hat{\mu}_A(x), \hat{\mu}_A(y), [0.5,0.5]].$$

Since A is an interval valued fuzzy subgroup of G, it then follows that

$$\hat{\mu}_A(xy^{-1}) \geq r\min[\hat{t}, [0.5,0.5]]$$

since otherwise, it will imply that

$$\hat{\mu}_A(xy^{-1}) < r\min[\hat{t}, [0.5,0.5]]$$

which means that

$$U(x,t) \in \bar{v}q\ \mu\ or\ U(y,t) \in \bar{v}q\ \mu,$$

a contradiction.
If

$$M(\hat{t},[0.5,0.5]) = \hat{t},\ then\ xy^{-1} \in \hat{A}_{\hat{t}},$$

and if

$$M(\hat{t},[0.5,0.5]) = [0.5,0.5],\ then\ \hat{\mu}_A(xy^{-1}) + \hat{t} > [1,1]$$

and hence,

$$xy^{-1} \in \hat{A}_{\hat{t}}.$$

So,$\hat{A}_{\hat{t}}$ is a subgroup of G.
Conversely, let A be a fuzzy subset of G such that $\hat{A}_{\hat{t}}$ is a subgroup of G for every $\hat{t} \in (0,1]$. If possible,let

$$\hat{\mu}_A(xy^{-1}) < \hat{t} < M[\hat{\mu}_A(x), \hat{\mu}_A(y), [0.5,0.5]]$$

for some $\hat{t} \in (\hat{0}, [0.5, 0.5])$. Then $x, y \in \hat{A}_{\hat{t}}$. That is

$$\hat{\mu}_A(xy^{-1}) \geq \hat{t} \textit{ or } \hat{\mu}_A(xy^{-1}) + \hat{t} > [1, 1]$$

which is a contradiction. Therefore

$$\hat{\mu}_A(xy^{-1}) \geq M[\hat{\mu}_A(x), \hat{\mu}_A(y), [0.5, 0.5]]$$

for every $x, y \in G$. So A is an $(\in, \in vq)$ fuzzy subgroup of G

Definition (3.7) An interval valued fuzzy subset A of a group G is said to be an interval valued $(\in, \in vq)$-fuzzy normal subgroup of G if $\forall\ x, y \ \in \ G$ and $a, b \ \in \ D[0, 1]$,
$U(x, \hat{a}) \ \in \ A) \Longrightarrow U(yxy^{-1}) \ \in vq \ A$

Theorem (3.8) An interval valued fuzzy subset A of a group G is an interval valued $(\in, \in vq)$-fuzzy normal subgroup of G if and only if $\hat{A}_{\hat{t}}$ is an interval valued $(\in, \in vq)$-fuzzy normal subgroup of $G \ \forall\ \hat{t} \ \in \ D[0, 1]$.

Proof Let A an interval valued $(\in, \in vq)$-fuzzy normal subgroup of G. Then, $\hat{A}_{\hat{t}}$ is a subgroup of G by Theorem 3.2. Let $x \ \in \ \hat{A}_{\hat{t}}$,so that $\hat{\mu}_A(x) \ \geq \ \hat{t}$, then $U(x, \hat{t}) \ \in \ A$. And since A is fuzzy normal,

$$U(x, \hat{t}) \ \in \ A \ \Longrightarrow \ U(yxy^{-1}, \hat{t}) \ \in vq \ A$$

for some $y \ \in \ G$. So that

$$\hat{\mu}_A(yxy^{-1}) \ \geq \ r\min[\hat{\mu}_A(x), [0.5, 0.5]]$$

which shows that $U(yxy^{-1}, \hat{t}) \ \in \ A$. That is $yxy^{-1} \ \in \ \hat{A}_{\hat{t}}$. Hence, $\hat{A}_{\hat{t}}$ is an interval valued $(\in, \in vq)$-fuzzy normal subgroup of $G \ \forall\ \hat{t} \ \in \ D[0, 1]$.
Conversely, let A be an interval valued fuzzy subset such that $\hat{A}_{\hat{t}}$ is an interval valued $(\in, \in vq)$-fuzzy normal subgroup of $G \ \forall\ \hat{t} \ \in \ D[0, 1]$. Then A is an interval valued $(\in, \in vq)$ -fuzzy subgroup of G by Theorem 3.2. We next show that A is normal. For A to be normal, we must have

$$U(x, \hat{a}) \ \in \ A \ \Rightarrow \ U(yxy^{-1}, \hat{a}) \ \in vq \ A$$

which is equivalent to

$$\hat{\mu}_A(yxy^{-1}) \ \geq \ r\min[\hat{\mu}_A(x), [0.5, 0.5]].$$

suppose the contrary, that is, suppose that

$$\hat{\mu}_A(yxy^{-1}) \ < \ r\min[\hat{\mu}_A(x), [0.5, 0.5]].$$

for some $x, y \ G$. If $\hat{t}$ is chosen such that

$$\hat{\mu}_A(yxy^{-1}) \ < \ \hat{t} \ < \ r\min[\hat{\mu}_A(x), [0.5, 0.5]],$$

then it follows that $\hat{\mu}_A(x) > \hat{t}$, which implies that $x \in \hat{A}_{\hat{t}}$. But $yxy^{-1} \bar{\in} \hat{A}_{\hat{t}}$. that $U(yxy^{-1}, \hat{t}) \in A$. That is a contradiction since $\hat{A}_t$ is fuzzy normal. Hence, it follows that

$$\hat{\mu}_A(yxy^{-1}) \geq r\min[\hat{\mu}_A(x), [0.5, 0.5]],$$

and the result follows accordingly.

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