Nigerian Journal of Mathematics and Applications
Volume **36**, (2026), 81 − 91. *Printed by Unilorin press*

http://www.njmaman.com

On Specification of Almon Distribued Lag Model in the Presence of Multicollinearity

T. K. Ajetunmobi and O. K. Agunloye*

Abstract

In Almon distributed lag modeling, presence of multicollinearity, choice of appropriate approximating polynomial of suitable degree and choice of optimal truncation lag-length required for the implementation of Almon distributed lag model constitute three major problems confronting Almon distributed lag modeling.Given the fact that the choice of appropriate approximating polynomial of suitable degree and optimal truncation lag-length can significantly reduce problem of multicollinearity in Almon model, we employ variance inflation factor as an alternative but credible criterion to choose appropriate approximating polynomial of suitable degree as well as circumventing the problem of under-fitting and over-fitting of truncation lag-length associated with standard information criteria.For empirical illustration, several Almon models were fitted to Nigerian inflation data against Naira-based exchange rate data. From the results of data analysis,we showed that approximating polynomial of order 2 is the most appropriate and the lag-length of Almon model with the lowest variance inflation factor was selected as the optimal truncation lag-length that guarantee minimal multicollinearity.

Received: 10/05/2026, Accepted: 02/06/2026, Revised: 21/06/2026. * Corresponding author.
2015 *Mathematics Subject Classification.* 62J05, 62J07 & 62P20.
Keywords and phrases. Almon Distributed lag Model, Multicollinearity, Truncation lag-length, Approximating Polynomial
Department of Statistics, Obafemi Awolowo University, Ile Ife 220005, Nigeria
E-mails: kagunloye@oauife.edu.ng
ORCID of the corresponding author: 0000-0002-0496-5382

1. Introduction and Preliminaries

Almon in [2] introduced a distributed lag technique that represents an alternative approach to modeling distributed lag models. Almon technique imposes a polynomial structure on lag coefficients to address the problem of multicollinearity in finite distributed lag model. It reduces the parameter space by assuming that lags follow a low-degree polynomial, enabling estimation in time series data where effects persist over time. A general problem with Almon technique is that the truncation lag-length and correct polynomial degree are not known a priori. The effects of misspecifications of these crucial parameters are potentially serious in terms of biased estimation and reliability of inference arising therefrom. This issue is discussed in detail in [7, 10, 11, 13]. [5] showed that approximating polynomial of order two is the best for fitting Almon distributed lag models. [9] introduced infinite truncation lag-length as a modification to the Almon distributed lag model to address the problem of multicollinearity in distributed lag models. This modification addresses the limitation of the standard Almon technique that assumes a finite truncation lag length. [3] demonstrated that the optimal order of approximating polynomial depends on several factors such as the lag weight, the number of lags assumed in the model, the sample size, the ratio of the variance of the dependent variable to that of the error term, and the degree of autocorrelation of the independent variable. [7] showed that the choice of appropriate optimal truncation lag length and approximating polynomial of suitable degree minimizes the residual variance. [10] compared the efficiency of Almon estimator and the ordinary least squares (OLS) method for distributed lag models and showed that the Almon estimator is more efficient than the OLS approach. [6] argued that Almon distributed lag model imposes a shape on lag distribution which reduces problem of multicollinearity in Almon distributed lag modeling. Asides the problem of multicollinearity, distributed lag model also suffer from heteroscedasticity. [4] proposed an adaptive version of Almon Two-Parameter Estimator (ATPE) which is more effective than the ATPE when the unknown form of heteroscedasticity is present in distributed lag model. [14] proposed a new bootstrapped Liu shrinkage estimator to address the problem multicollinearity in the Almon distributed lag model.

2. Methodology

Almon Distributed Lag Model Consider a finite distributed lag model of the form as defined in (1) below:

$$Y_t = \sum_{j=0}^{k} \beta_j X_{t-j} + u_t \tag{1}$$

where:
Y_t is the dependent variable
X_{t-j} are the lagged values of independent variable x_t
β_j represents the effect of X_t lagged by period j on Y_t,
u_t is the error term assumed to be normally distributed with mean zero and variance σ^2.
[2] assumed that the model parameters of (1) can be approximated by a suitable-degree polynomial in i, which has $k+1$ coefficients of β that can be specified as:

$$\beta_i = \sum_{j=0}^{q} \alpha_j i^j \qquad for \qquad i = 0, ..., k \tag{2}$$

For notational convenience, zero to the zero power is defined to be 1. Let β_i be modelled as a polynomial function of order q

$$\beta_i = \alpha_0 + \alpha_1 i + \alpha_2 i^2 + ... + \alpha_q i^q \tag{3}$$

let $\Pi = (i^j)$ and let $\alpha' = (\alpha_0, \alpha_1, ..., \alpha_q)$.
(3) could be written as a system of equations as shown below:

$$\begin{array}{ll}
\text{when } i = 0 & \beta_0 = \alpha_0 \\
i = 1 & \beta_1 = \alpha_0 + \alpha_1 + \alpha_2 + ... + \alpha_q \\
i = 2 & \beta_2 = 2^0\alpha_0 + 2^1\alpha_1 + 2^2\alpha_2 + ... + 2^q\alpha_q \\
\vdots & \vdots \qquad \vdots \\
i = k & \beta_k = k^0\alpha_0 + k^1\alpha_1 + k^2\alpha_2 + ... + k^q\alpha_q
\end{array}$$

the above system of equations can be transformed into matrix form as shown in (4)

$$\beta = \Pi\alpha \tag{4}$$

$$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 4 & \cdots & 2^q \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & k & k^2 & \cdots & k^q \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_q \end{pmatrix} \tag{5}$$

By putting (2) in (1), we have:

$$y_t = \sum_{i=0}^{k} \sum_{j=0}^{q} \alpha_j i^j x_{t-i} + u_t = \sum_{j=0}^{q} \alpha_j z_j + u_t, \tag{6}$$

where $z_j = \sum_{i=0}^{k} i^j x_{t-i}$. Given a set of T observations $(x_1, y_1), ..., (x_T, y_T)$, we can form variables $z_{0t}, ..., z_{qt}$ for $t = 1, 2, ..., T$. Then, by using the method of ordinary least squares to regress y_t on these variables, we can find estimates of the polynomial parameters $\alpha_0, \cdots, \alpha_q$. By substituting these estimates in (2), we can find estimates of lag coefficients $\beta_0, \beta_1, \cdots, \beta_k$. (1) can be written in matrix form as:

$$y = X\beta + u \tag{7}$$

then, by setting $\beta = \Pi\alpha$ in (7) we have

$$y = X\Pi\alpha + u = Z\alpha + u \tag{8}$$

From the ordinary least square estimate

$$\hat{\alpha} = (Z'Z)^{-1} Z'y \tag{9}$$

we could find

$$\hat{\beta} = \Pi\hat{\alpha} \tag{10}$$

(10) is called Almon estimator of β. A well.known problem with Almon technique is the choice of optimal truncation lag length and the degree of approximating polynomial because these two model specification decisions have far-reaching impact on the outcome of Almon distributed lag modeling

3. Data Analysis

Stationarity Test for Nigeria inflation Rate and Exchange Rate Series. Unit root test was carried out using augmented Dickey-Fuller(ADF) unit root test proposed by Said and Dickey(1984) to check if there is presence of unit root in the series. This test is designed to determine whether a time series is stable around its level (which is called trend-stationary) or stable around the differences in its levels (which is called difference-stationary). To perform the ADF test, we need to choose the appropriate specification of the regression equation and the number of lagged differences to include in such specification. The choice of a particular ADF specification is determined by whether we expect the time series to have a constant mean, a linear trend or neither. The ADF test considered in this paper has both constant mean and a linear trend. The truncation lag-length was determined by using information criteria, such as Akaike Information Criterion (AIC) proposed by [1] Akaike(1973) and Bayesian Information Criterion (BIC) proposed by [12]. [8] proposed ADF regression of the form:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \delta_2 \Delta y_{t-2} + ... + +\delta_p \Delta y_{t-p} + \epsilon_t \tag{11}$$

where:
Δy_t is the first difference of the time series y
α is a constant term
,βt is a linear trend term ,

y_{t-1} is the first lagged value of y_t, and Δy_{t-p} are the lagged differences of y_t up to p lags,
The null and alternative hypotheses of augmented Dickey-Fuller(ADF) test of (11) are:
$H_0 : \gamma = 0$ - The series is non-stationary
$H_1 : \gamma < 0$ - The series is stationary.

Table 1. Unit Root Test at level

Variable	Test Statistic	Critical Value at 5%	Decision
Nigeria Inflation rate	-2.510945	-3.433525	Accept H_0
Naira-Dollar Exchange rate	-1.605437	-3.432452	Accept H_0
Naira-Pound Exchange rate	-1.545523	-3.432115	Accept H_0
Naira-Euro Exchange rate	-1.940095	-3.432115	Accept H_0

From table 1, the ADF test statistic value is greater than critical value at 5% level of significance for each of the four variables being tested for the presence of unit root. Consequently, we fail to reject the null hypothesis and we conclude that Nigeria inflation rate, Naira-Dollar Exchange rate, Naira-Pound Exchange rate and Naira-Euro Exchange rate are non-stationary at level.

Table 2. Unit Root Test at first difference

Variable	Test Statistic	Critical Value at 5%	Decision
Nigeria Inflation rate	-4.842602	-3.433525	Reject H_0
Naira-Dollar Exchange rate	-6.980288	-3.432452	Reject H_0
Naira-Pound Exchange rate	-11.04970	-3.432115	Reject H_0
Naira-Euro Exchange rate	-9.552029	-3.432115	Reject H_0

From table 2, the ADF test statistic value is less than critical value at 5% level of significance for each of the four variables being tested for the presence of unit root at first difference. Consequently, we reject the null hypothesis and we conclude that Nigeria inflation rate, Naira-Dollar Exchange rate, Naira-Pound Exchange rate and Naira-Euro Exchange rate are stationary at first difference.

3.1. **Almon Models when Lag-Length is 2 and Degree of Approximating Polynomial is 2.** Three fitted Almon distributed lag models are presented hereunder when lag-length is 2 and degree of approximating polynomial is 2

Almon Model for Nigeria Inflation Rate against Naira-Dollar Exchange Rate:

$$y_t = 0.0358x_t + 0.00763x_{t-1} + 0.0278x_{t-2} + u_t \tag{12}$$

Almon Model for Nigeria Inflation Rate against Naira-Pound Exchange Rate:

$$y_t = 0.0536x_t - 0.00198x_{t-1} + 0.0115x_{t-2} + u_t \tag{13}$$

Almon Model for Nigeria Inflation Rate against Naira-Euro Exchange Rate:

$$y_t = 0.0126x_t - 0.00576x_{t-1} + 0.00797x_{t-2} + u_t \tag{14}$$

3.2. **Almon Models when Lag-Length is 3 and Degree of Approximating Polynomial is 2.** Three fitted Almon distributed lag models are presented hereunder when lag-length is 3 and degree of approximating polynomial is 2

Almon Model for Nigeria Inflation Rate against Naira-Dollar Exchange Rate:

$$y_t = 0.0348x_t + 0.0096x_{t-1} - 0.0086x_{t-2} - 0.0198x_{t-3} + u_t \tag{15}$$

Almon Model for Nigeria Inflation Rate against Naira-Pound Exchange Rate:

$$y_t = 0.01110x_t - 0.00749x_{t-1} - 0.00536x_{t-2} + 0.01750x_{t-3} + u_t \tag{16}$$

Almon Model for Nigeria Inflation Rate against Naira-Euro Exchange Rate:

$$y_t = 0.01590x_t - 0.00390x_{t-1} - 0.00602x_{t-2} + 0.00956x_{t-3} + u_t \tag{17}$$

3.3. **Almon Models when Lag-Length is 4 and Degree of Approximating Polynomial is 3.** Three fitted Almon distributed lag models are presented hereunder when lag-length is 4 and degree of approximating polynomial is 3:

Almon Model for Nigeria Inflation Rate against Naira-Dollar Exchange Rate:

$$y_t = 0.0301x_t + 0.0130x_{t-1} - 0.00042x_{t-2} - 0.0103x_{t-3} - 0.0165x_{t-4} + u_t \tag{18}$$

Almon Model for Nigeria Inflation Rate against Naira-Pound Exchange Rate:

$$y_t = 0.00909x_t - 0.00173x_{t-1} - 0.00467x_{t-2} + 0.000298x_{t-3} + 0.0132x_{t-4} + u_t \tag{19}$$

Almon Model for Nigeria Inflation Rate against Naira-Euro Exchange Rate:

$$y_t = 0.00133x_t + 0.0022x_{t-1} - 0.00288x_{t-2} - 0.00190x_{t-3} + 0.00514x_{t-4} + u_t \tag{20}$$

3.4. **Determination of Optimal Truncation Lag-length.** Most contemporary literature evaluates the optimal combination of truncation lag-length and approximating polynomial degree via standard model selection criteria like Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) or minimized Mean Squared Error (MSE) through Monte Carlo simulations rather than fixed ad-hoc rules.In this paper,two standard information criteria such as Akaike information criterion and Bayesian information criterion were used to determine optimal truncation lag-length and degree of approximating polynomial for the fitted Almon distributed lag models. Following the result presented in tables 3-5 below, Almon distributed lag model with lag-length 2 has the smallest standard error when compared with other Almon distributed lag models fitted with higher lag-length. Similarly, based on the results presented in tables 3-5 below, Almon distributed lag model with lag-length 2 has the smallest value of the information criteria when compared with other Almon distributed lag models fitted with higher lag-length. Hence, lag-length 2 is the optimal truncation lag-length.

Table 3. Evaluation of Different Almon Models Fitted for Nigeria Inflation Rate
against Naira-Dollar Exchange Rate

Lag-Length	Polynomial Order	AIC	BIC	S.E
2	2	1117.76	1134.33	2.852***
3	2	1102.6	1119.14	3.131
3	3	1104.43	1124.28	3.215
4	2	1094.28**	1110.79**	3.317
4	3	1096.07	1115.89	3.416
4	4	1098.05	1121.17	3.643

Table 4. Evaluation of Different Almon Models Fitted for Nigeria Inflation Rate against Naira-Pound Exchange Rate

Lag-Length	Polynomial Order	AIC	BIC	S.E.
2	2	1123.79	1140.35	2.697***
3	2	1109.566	1126.11	3.401
3	3	1111.57	1131.42	3.619
4	2	1101.97**	1118.49**	3.588
4	3	1103.91	1123.73	3.716
4	4	1105.90	1129.03	3.715

Table 5. Evaluation of Different Almon Models Fitted for Nigeria Inflation Rate
against Naira-Euro Exchange Rate

Lag-Length	Polynomial Order	AIC	BIC	S.E.
2	2	1122.12	1122.12	2.771***
3	2	1107.47	1124.02	3.234
3	3	1109.46	1129.31	3.522
4	2	1099.53**	1116.046**	3.359
4	3	1101.40	1121.22	3.586
4	4	1103.40	1126.53	3.699

3.5. **Determination of Appropriate Approximating Polynomial Required For the Implementation of Almon Distributed Lag Model.** The Variance Inflation Factor (VIF) and Tolerance Limit(ToL) are two standard measures of multicollinearity used to evaluate the effects of the choice of approximating polynomial on the level of multicollinearity in Almon polynomial distributed lag models. Almon distributed lag models were fitted with approximating polynomials of different orders.The results presented in tables 6 and 7 below showed that the VIF increases as the order of approximating polynomial increases for the three Almon models labelled as a, b and c.This reveal that higher order approximating polynomial escalates problem of multicollinearity as the degree of the polynomial increases, the values of the Variance Inflation Factor (VIF) also rise. This pattern demonstrates that multicollinearity intensifies in direct proportion to the polynomial degree. In other words, higher-order Almon polynomials tend to exacerbate multicollinearity, leading to inflated VIF values and thereby making the estimation of the coefficients more unreliable. Lower-degree approximating polynomials, conversely, maintain more manageable levels of multicollinearity.

Table 6. Variance Inflation Factor(VIF) and Tolerance Limit of Almon Models under Different Order of Approximating Polynomials

a. b. c.

Order of Aproximating Polynomial	VIF	Tol.	VIF	Tol.	VIF	Tol
2	1.155**	0.866**	1.1157**	0.8963**	1.1273**	0.8871**
3	1.156	0.865	1.1160	0.8961	1.1274	0.8870
4	1.156	0.865	1.1217	0.8915	1.1362	0.8801

Table 7. Variance Inflation Factor(VIF) and Tolerance Limit of Almon Models under Different Order of Approximating Polynomials

a. b. c.

Order of Approximating Polynomial	VIF	Tol.	VIF	Tol.	VIF	Tol
2	1.1650**	0.8584**	1.1213**	0.8918**	1.1349**	0.8811**
3	1.1661	0.8576	1.1216	0.8916	1.1357	0.8805
4	1.1663	0.8574	1.1217	0.8915	1.1362	0.8801

Remark:
a: Almon nodel for Nigeria Inflation Rate against Naira-Dollar Exchange Rate
b: Almon nodel for Nigeria Inflation Rate against Naira-Pound Exchange Rate
c: Almon nodel for Nigeria Inflation Rate against Naira-Euro Exchange Rate

Conclusion: This paper examined model specification issues concerning Almon distributed lag model.Our findings showed that optimal truncation lag-length for Almon model is 2, approximating polynomial of order 2 is the best for Almon model as higher order approximating polynomial escalates problem of multicollinearity and the problem of multicollinearity is evidently minimal in Almon distributed lag model as the variance inflation factor is not above 2 in all cases considered.

Acknowledgement: The authors are grateful to Obafemi Awolowo University Ile-Ife, Nigeria for the supports they received during the compilation of this work.

Competing interests: The manuscript was read and approved by all the authors. They therefore declare that there is no conflicts of interest.

Funding: The Authors received no financial support for the research, authorship, and/or publication of this article.

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