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Influence of Variable Viscosity and Slip Velocity on Unsteady Magnetohydrodynamic (Mhd) Flow of Blood Through a Stenosed Artery

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Abstract

In this articles, influence of variable viscosity and slip velocity on unsteady MHD flow of blood through a stenosed artery are investigated, The blood viscosity is assumed to varying radially with hematocrit throughout the region of the artery. The analytical expressions for the flow characteristics are obtained using Galerkin's and Fourth order Runge-Kutta methods. It was revealed from the results that, increase in Hematocrit parameter lead to increase in shear stress and resistance to flow but decreases the flow velocity and volume flow rate. Also, velocity profile and volume flow rate increases significantly with increase values of the slip velocity but reduces the resistance to flow. Magnetic field parameter increases with resistance to flow but slightly decreases the velocity profile of the blood flow. Other parameters that could have a positive or negative effects on the flow characteristics are Reynolds number, Pressure gradient and Hartmann number.

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Nomenclatures
u - Fluid velocity $\overline{u}$- Dimensionless fluid velocity
t - Time component $\overline{t}$ - Dimensionless time component
r - Radial distance $\overline{y}$ - Dimensionless radial distance
z - Axial distance u_s - Slip velocity
R_0 - Radius of the normal artery β_0 - Magnetic Field Strength
R(z) - Radius of the artery in a stenotic region σ - Electrical Conductivity
ψ - Resistance to flow K - Thermal conductivity
Q - Volumetric flow rate τ_s - Shear Stress
? - Maximum height of the stenosis L - Length of the stenosis
H_P = Hematocrit parameter H_N = Hartmann number
h(y) = Hematocrit at a distance r μ_0 = Viscosity coefficient for plasma
β = A constant whose value for blood equal 2.5 R_N = Reynold number
μ(y) = Coefficient of viscosity of blood at radial distance
P_G = Pressure gradient
u_0 = Dimensionless Slip velocity
M_P = Magnetic parameter

1. Introduction and Preliminaries

Cardiovascular diseases mainly atherosclerosis (medical called stenosis) which are diseases of the heart and blood vessel occurred as a result of artificial occlusion of the arteries and is one of the leading cause of death globally [16]. When artificial occlusion occurred as a result of stenotic obstruction, it is not only restrict the regular flow of blood but also lead to hardening and thickening of the arterial wall.

Magnetohydrodynamics (MHD) is a branch of science that study the dynamics behaviour of electrically conducting fluids in the presence of magnetic field without taking into consideration the magnetization effects of the fluid. MHD has various application such as bioengineering and medical sciences [14]. These areas of study had captivated the attention of many researchers [1], [2], [3] and [4]. All these researchers assuming constant viscosity of blood flow.

However, in a real physiological system, the blood viscosity is not constant, it may vary with hematocrit or depends on temperature and pressure of the blood flow [17]. Hematocrit effect on the axisymmetric blood flow through stenosed arteries has been investigated by [15]. [21] studied mathematical model of blood flow through a tapered artery with mild stenosis and hematocrit. Some of the other researchers that considered variable viscosity in their studies includes; [5], [6], [9], [10], [11], [18] and [19].

The aforementioned researchers considered only the steady fluid flow models, that is the flow in which the conditions at any point of the fluid flow were treated

as time independent. However, studies in which the conditions at any point of the fluid flow depend on time had received scanty attention because the equation governing such fluid flow have time derivative which renders the equation cumbersome to handle. Some of the researchers that considered unsteady fluid flow without considering the combined effects of slip velocity, magnetic field and hematocrit are [7], [8] and [20].
Motivated by the above analysis, this paper present the influence of variable viscosity and slip velocity on unsteady magnetohydrodynamics (MHD) flow of blood through a stenosed artery.

2. Mathematical Models

The equation governing the unsteady MHD blood flow as obtained by [13] is given as

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-\mu\frac{u}{K}-\sigma B_o^2 u=0 \tag{1}$$

The associated slip conditions to (1) are $u=u_s$ at $r=R(z)$

$$\frac{\partial u}{\partial r}=0 \quad atr=0 \tag{2}$$

Since one is considering variable viscosity dependence hematocrit, μ is replaced by $\mu(r)$ in (1) to obtain

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial z}+\mu(r)\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-\mu(r)\frac{u}{K}-\sigma B_o^2 u \tag{3}$$

By dividing equation (3) all through by ρ, one obtain

$$\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial z}\frac{1}{\rho}+\frac{\mu(r)}{\rho}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-\frac{\mu(r)\,u}{K\rho}-\frac{\sigma B_o^2 u}{\rho} \tag{4}$$

The second term in the RHS of equation (4) can be re-written as

$$\frac{\mu(r)}{\rho}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)=\frac{\mu(r)}{\rho}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{5}$$

According to Einstein formula, the variable viscosity $\mu(r)$ of blood is taken to be

$$(r)=\ {}_0(1+\beta h(r)) \tag{6}$$

and the hematocrit $h(r)$ is described by [12].

$$h\left(r\right)=H\left(1-\left(\frac{r}{R_0}\right)^n\right),n¿2 \tag{7}$$

By putting (5), (6) and (7) into (4), after simplifying, one obtain

$$\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial z}\frac{1}{\rho}+\frac{\mu_o}{\rho}\left[1+H_p\left(1-\left(\frac{r}{R_o}\right)^n\right)\right]\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$$

$$\frac{\mu_o u}{K\rho}\left[1+H_p\left(1-\left(\frac{r}{R_o}\right)^n\right)\right]-\frac{\sigma B_o^2 u}{\rho} \tag{8}$$

where H_p is the hematocrit parameter and n is the shape of the profile. Equation (8) is the equation governing the unsteady MHD flow of blood through a stenosed artery with hematocrit and slip velocity.
To solve equation (8) above, one introduce the dimensionless variables as follows

$$u=\frac{\overline{u}d}{t_o},\qquad r=\overline{r}R_o,\qquad t=\overline{t}t_o \tag{9}$$

Putting (9) into (8) and simplified to obtain

$$\frac{\partial\overline{u}}{\partial\overline{t}}=-\frac{t_o^2}{d\rho}\frac{\partial p}{\partial z}+\frac{t_o\mu_o}{\rho R_o^2}\left[1+H_p\left(1-\overline{r}^n\right)\right]\frac{1}{\overline{r}}\frac{\partial}{\partial\overline{r}}\left(\overline{r}\frac{\partial\overline{u}}{\partial\overline{r}}\right) \tag{10}$$

$$-\frac{\mu_o t_o}{K\rho}\overline{u}\left[1+H_p\left(1-\overline{r}^n\right)\right]-\frac{\sigma B_o^2 t_o}{\rho}\overline{u}$$

Equation (10) can be re-written as

$$\frac{\partial\overline{u}}{\partial\overline{t}}=P_G+\frac{1}{R_N}\left[1+H_p\left(1-\overline{r}^n\right)\right]\frac{1}{\overline{r}}\frac{\partial}{\partial\overline{r}}\left(\overline{r}\frac{\partial\overline{u}}{\partial\overline{r}}\right)$$

$$H_N\overline{u}\left[1+H_p\left(1-\overline{r}^n\right)\right]-Mp\overline{u} \tag{11}$$

where

$$R_N=\text{Reynold number}=\frac{\rho R_o^2}{t_o\mu_o}$$

$$H_N=\text{Hartmann number}=\frac{\mu_o t_o}{K\rho} \tag{12}$$

$$Mp=\text{Magnetic field parameter}=\frac{\sigma B_o^2 t_o}{\rho}$$

$$P_G=\text{Pressure gradient}=-\frac{t_o^2}{d\rho}\frac{\partial p}{\partial z}$$

Equation (11) is the dimensionless equation governing the unsteady MHD flow of blood with variable viscosity and slip velocity.
By using the dimensionless variables in (9) and the boundary conditions in (2), the corresponding dimensionless slip conditions to equation (11) can be written as

$$
\begin{aligned}
\overline{u} = u_o \quad at \quad \overline{r} = \tfrac{R(Z)}{R_o} = R_Z \\
\tfrac{\partial \overline{u}}{\partial \overline{r}} = 0 \quad at \quad \overline{r} = 0
\end{aligned}
\tag{13}
$$

3. Solution of the Transformed Equation

In order to obtain solution to (11) using Galerkin's Weighted Residual method, one assume a solution of the form

$$
\overline{u}\left(\overline{y}\right) = u_o\overline{y}^2 + C_o\left(\overline{t}\right)\left(1-\overline{y}^2\right) + C_2\left(\overline{t}\right)\overline{y}^2\left(1-\overline{y}^2\right) \tag{14}
$$

The residue for equation (11) can be written as:

$$
\begin{aligned}
RR2\left(C_o, C_2, \overline{r}\right) = \frac{\partial \overline{u}}{\partial \overline{t}} - P_G - \frac{1}{R_N}\left[1 + H_p\left(1-\overline{r}^n\right)\right]\frac{1}{\overline{r}}\frac{\partial}{\partial \overline{r}}\left(\overline{r}\frac{\partial \overline{u}}{\partial \overline{r}}\right) \\
+ H_N\overline{u}\left[1 + H_p\left(1-\overline{r}^n\right)\right] + \ Mp\overline{u}
\end{aligned}
\tag{15}
$$

Taking the shape of the profile $(n = 2)$ and using the transformation $\overline{y} = \frac{\overline{r}}{R_Z}$, substituting (14) into (15) to obtains

$$
\begin{aligned}
RR2\left(C_o, C_2, \overline{y}\right) = \dot{C}_o\left(\overline{t}\right)\left(1-\overline{y}^2\right) + \dot{C}_2\left(\overline{t}\right)\overline{y}^2\left(1-\overline{y}^2\right) - P_G \\
-\frac{1}{R_N}\left[1 + H_p\left(1-\overline{y}^n\right)\right]4\left(u_o - C_o\left(\overline{t}\right) + C_2\left(\overline{t}\right) - 4C_2\left(\overline{t}\right)\overline{y}^2\right) \\
+H_N\left(u_o\overline{y}^2 + C_o\left(\overline{t}\right)\left(1-\overline{y}^2\right) + C_2\left(\overline{t}\right)\overline{y}^2\left(1-\overline{y}^2\right)\right)\left[1 + H_p\left(1-\overline{y}^n\right)\right] \\
+Mp\left(u_o\overline{y}^2 + C_o\left(\overline{t}\right)\left(1-\overline{y}^2\right) + C_2\left(\overline{t}\right)\overline{y}^2\left(1-\overline{y}^2\right)\right)
\end{aligned}
\tag{16}
$$

Simplifying (16) to obtains

$$
\begin{aligned}
RR2\left(C_o, C_2, \overline{y}\right) = \dot{C}_o\left(\overline{t}\right)\left(1-\overline{y}^2\right) + \dot{C}_2\left(\overline{t}\right)\overline{y}^2\left(1-\overline{y}^2\right) - P_G \\
-\frac{1}{R_N}\left[1 + \ H_p\left(1-\overline{y}^n\right)\right]\left(4u_o - 4C_o\left(\overline{t}\right) + 4C_2\left(\overline{t}\right) - 16C_2\left(\overline{t}\right)\overline{y}^2\right) \\
+H_N\left(u_o\overline{y}^2 + C_o\left(\overline{t}\right)\left(1-\overline{y}^2\right) + C_2\left(\overline{t}\right)\overline{y}^2\left(1-\overline{y}^2\right)\right)\left[1 + H_p\left(1-\overline{y}^n\right)\right] \\
+Mp\left(u_o\overline{y}^2 + C_o\left(\overline{t}\right)\left(1-\overline{y}^2\right) + C_2\left(\overline{t}\right)\overline{y}^2\left(1-\overline{y}^2\right)\right)
\end{aligned}
\tag{17}
$$

Simplifying (17) further to obtain

$$
\begin{aligned}
RR2\left(C_o, C_2, \overline{y}\right) = \frac{4}{R_N} C_o\left(\bar{t}\right) - \frac{4}{R_N} C_2\left(\bar{t}\right) + \frac{16 C_2\left(\bar{t}\right)\overline{y}^2}{R_N} + \frac{4 C_o\left(\bar{t}\right) H_p}{R_N} \\
- \frac{4 C_2\left(\bar{t}\right) H_p}{R_N} + \dot{C}_o\left(\bar{t}\right) - \frac{16 C_2\left(\bar{t}\right) H_p \overline{y}^4}{R_N} + \frac{20 C_2\left(\bar{t}\right) H_p \overline{y}^2}{R_N} - \frac{4 C_o\left(\bar{t}\right) H_p \overline{y}^2}{R_N} \\
+ H_N C_o\left(\bar{t}\right) H_p \overline{y}^2 + H_N C_o\left(\bar{t}\right) H_p \overline{y}^4 - 2 H_N C_o\left(\bar{t}\right) H_p \overline{y}^2 + H_N C_2\left(\bar{t}\right) H_p \overline{y}^6 \\
-2 H_N C_2\left(\bar{t}\right) H_p \overline{y}^4 + H_N C_2\left(\bar{t}\right) H_p \overline{y}^2 - \dot{C}_o\left(\bar{t}\right)\overline{y}^2 - \dot{C}_2\left(\bar{t}\right)\overline{y}^4 + H_N C_o\left(\bar{t}\right) \\
+ M_p C_o\left(\bar{t}\right) - M_p C_2\left(\bar{t}\right)\overline{y}^4 + M_p C_2\left(\bar{t}\right)\overline{y}^2 - M_p C_o\left(\bar{t}\right)\overline{y}^2 - H_N C_2\left(\bar{t}\right)\overline{y}^4 \\
+ H_N C_2\left(\bar{t}\right)\overline{y}^2 - H_N C_o\left(\bar{t}\right)\overline{y}^2 + H_N H_p C_o\left(\bar{t}\right) + \dot{C}_2\left(\bar{t}\right)\overline{y}^2 + H_N u_o \overline{y}^2 \\
- \frac{4 u_o H_p}{R_N} - \frac{4 u_o}{R_N} - P_G + \frac{4 H_p u_o \overline{y}^2}{R_N} - H_N u_o H_p \overline{y}^4 + H_N u_o H_p \overline{y}^2
\end{aligned}
\tag{18}
$$

By differentiating (14) with respect to $C_o\left(\bar{t}\right)$ and $C_2\left(\bar{t}\right)$, one obtains the weight functions as $1-\overline{y}^2$ and $\overline{y}^2\left(1-\overline{y}^2\right)$ respectively.
By taking into account the orthogonality of the residue $RR\left(C_o\left(\bar{t}\right), C_2\left(\bar{t}\right), \overline{y}\right)$ with respect to the weight functions, one obtain after simplification the following system

$$
\begin{aligned}
\frac{8 C_o\left(\bar{t}\right)}{3 R_N} - \frac{8 C_2\left(\bar{t}\right)}{15} + \frac{32 H_p C_o\left(\bar{t}\right)}{15 R_N} - \frac{32 H_p C_2\left(\bar{t}\right)}{35 R_N} + \frac{8 \dot{C}_o\left(\bar{t}\right)}{15 R_N} + \frac{8 \dot{C}_2\left(\bar{t}\right)}{105} \\
+ \frac{8 H_N C_2\left(\bar{t}\right)}{105} + \frac{8 H_N C_2\left(\bar{t}\right)}{105} + \frac{8 M_p C_2\left(\bar{t}\right)}{105} + \frac{8 H_N C_o\left(\bar{t}\right)}{15} + \frac{8 M_p C_o\left(\bar{t}\right)}{15} \\
+ \frac{16 H_p H_N C_o\left(\bar{t}\right)}{35} + \frac{16 H_p H_N C_2\left(\bar{t}\right)}{315} - \frac{32 H_p u_o}{35 R_N} - \frac{8 u_o}{3 R_N} - \frac{2 P_G}{3} + \frac{2 M_p u_o}{15} \\
+ \frac{2 H_N u_o}{15} + \frac{8 H_p H_N u_o}{105} = 0
\end{aligned}
\tag{19}
$$

and

$$
\begin{aligned}
\frac{8 C_o\left(\bar{t}\right)}{3 R_N} + \frac{8 C_2\left(\bar{t}\right)}{21} + \frac{32 H_p C_o\left(\bar{t}\right)}{105 R_N} + \frac{32 H_p C_2\left(\bar{t}\right)}{315 R_N} + \frac{8 \dot{C}_o\left(\bar{t}\right)}{105 R_N} + \frac{8 \dot{C}_2\left(\bar{t}\right)}{315} \\
+ \frac{8 \dot{C}_2\left(\bar{t}\right)}{315} + \frac{8 H_N C_2\left(\bar{t}\right)}{315} + \frac{8 M_p C_2\left(\bar{t}\right)}{315} + \frac{8 H_N C_o\left(\bar{t}\right)}{105} + \frac{8 M_p C_o\left(\bar{t}\right)}{105} \\
+ \frac{16 H_N C_o\left(\bar{t}\right)}{315} + \frac{16 H_p H_N C_2\left(\bar{t}\right)}{1155} - \frac{32 H_p u_o}{105 R_N} - \frac{8 u_o}{15 R_N} \frac{2 P_G}{15} \\
+ \frac{2 M_p u_o}{35} + \frac{2 H_N u_o}{35} + \frac{8 H_p H_N u_o}{315} = 0
\end{aligned}
\tag{20}
$$

By substituting the appropriate values of P_G, H_p, H_N, M_p, R_N and u_o, into (19) and (20) and solved using fourth order Runge-Kutta with initial conditions $C_o\left(\bar{t}\right) = 0$, and $C_2\left(\bar{t}\right) = 0$, one obtains the values for $C_o\left(\bar{t}\right)$ and $C_2\left(\bar{t}\right)$ which when substituted into (14), one obtained velocity profile for the unsteady MHD blood flow through stenosed artery.
By simulating for different values of the parameters, one obtained different velocity profiles, volumetric flow rates, flow resistance and shear stress which are shows in the graphs below.

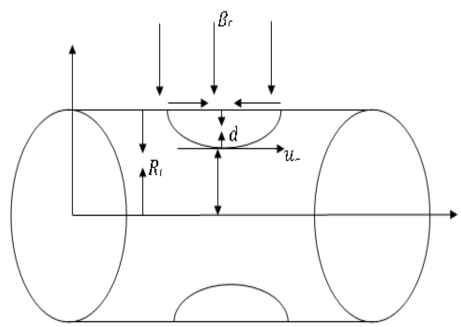

Figure 1: Geometry of the stenosis

The stenosis has been described by [12] and [22].

$$\frac{R(z)}{R_0} = 1 - \frac{\epsilon}{2R_0}\left[1 + \frac{cos\pi z}{L}\right], \tag{21}$$

$|z|\leq$L for R_0 in (21).

Volume Flow Rate
The volume flow rate denoted by Q is given by

$$Q = 2\pi \int_0^{R(z)} \overline{yu}\left(\overline{y}\right) d\overline{y} \tag{22}$$

Putting (14) into (22) and evaluate to obtain

$$Q = K \tag{23}$$

where K equals

$$K = 12\left[3V_0(R\,(z))^4 + c_0(t)\left(6(R\,(z))^2 - 3(R\,(z))^4\right) + c_2(t)\left(3(R\,(z))^4 - 2(R\,(z))^6\right)\right]$$

Shear Stress

The shear stress denoted by $\tau_S$is given as

$$\tau_s = \mu(r)\frac{\partial \overline{u}}{\partial \overline{y}}_{\overline{y}=R(z)} + 2\beta_3\left(\frac{\partial \overline{u}}{\partial \overline{y}}\right)^3_{\overline{y}=R(z)} \tag{24}$$

Simplified (24) to obtain

$$\begin{aligned} \tau_s = 2\mu(r)R(Z)\left(V_0 - c_0(t) + c_2(t) - 2R\,((Z))^2 c_2(t)\right) \\ +16R(Z)\beta_3\left(V_0 - c_0(t) + c_2(t) - 2(R\,(Z))^2 c_2(t)\right) \end{aligned} \tag{25}$$

Resistance to Flow

The resistance to flow can be denoted as Ψ and is given by

$$\Psi = \frac{-\frac{\partial \hat{P}}{\partial z}}{M} \tag{26}$$

where M equals

$$12\left[3V_0(R\,(z))^4 + c_0(t)\left(6(R\,(z))^2 - 3(R\,(z))^4\right) + c_2(t)\left(3(R\,(z))^4 - 2(R\,(z))^6\right)\right]$$

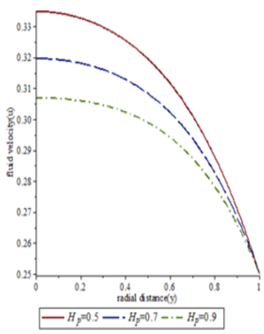

Figure 2: Variation of Velocity Profile of Unsteady MHD Blood Flow along the radial distance for different values of Hematocrit Parameter

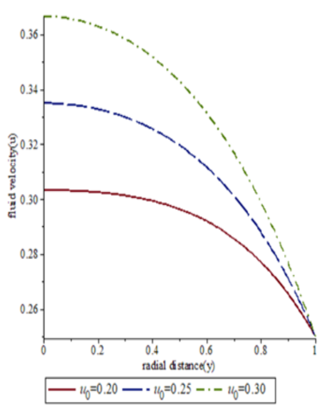

Figure 3: Variation of Velocity Profile of Unsteady MHD Blood Flow along the radial distance for different values of the slip velocity

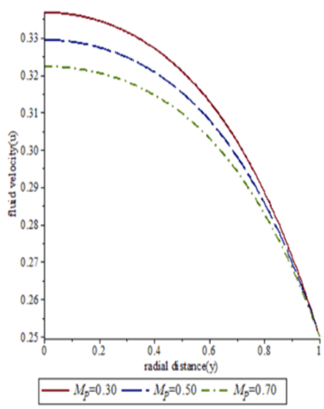

Figure 4: Variation of Velocity Profile of Unsteady MHD Blood Flow along the radial distance for different values of the Magnetic field Parameter

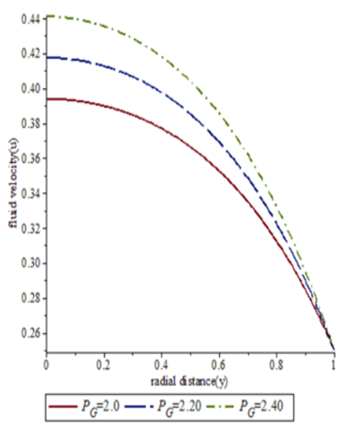

Figure 5: Variation of Velocity Profile of Unsteady MHD Blood Flow along the radial distance for different values of the Pressure gradient

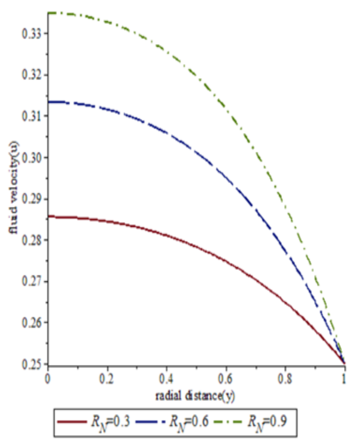

Figure 6: Variation of Velocity Profile of Unsteady MHD Blood Flow along the radial distance for different values of the Reynold number

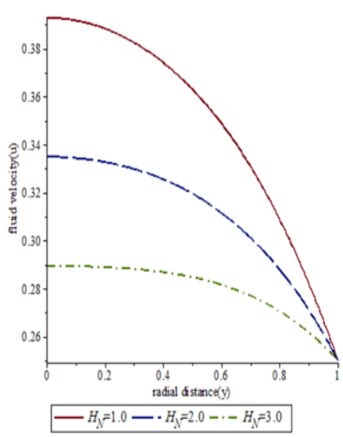

Figure 7: Variation of Velocity Profile of Unsteady MHD Blood Flow along the radial distance for different values of the Hartmann number

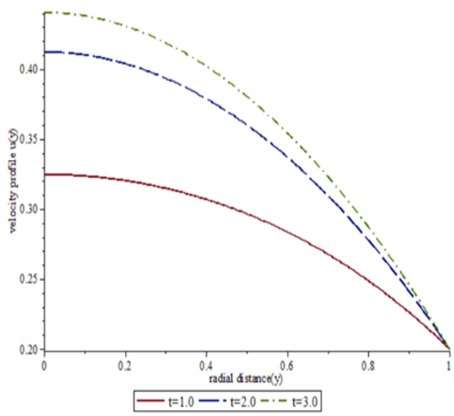

Figure 8: Variation of Velocity Profile of Unsteady MHD Blood Flow along radial distance at various time

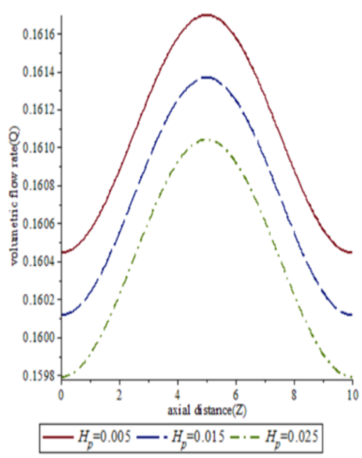

Figure 9: Variation of Volumetric Flow Rate of Unsteady MHD Blood Flow with increasing values of the Hematocrit Parameter in the entire arterial region along the axial direction

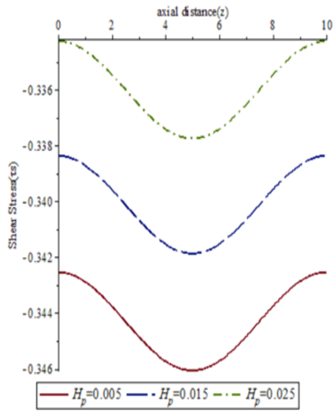

Figure 10: Variation of Wall Shear Stress of Unsteady MHD Blood Flow with increasing values of the Hematocrit Parameter in the entire arterial region along the axial direction

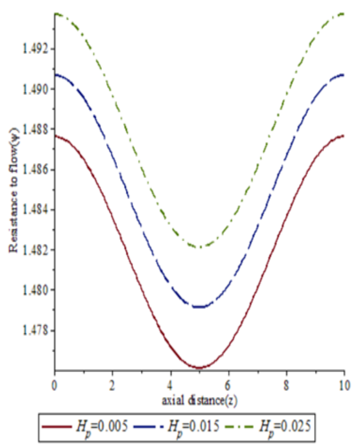

Figure 11: Variation of Resistance to Unsteady MHD Blood Flow with increasing values of the Hematocrit Parameter in the entire arterial region along the axial direction

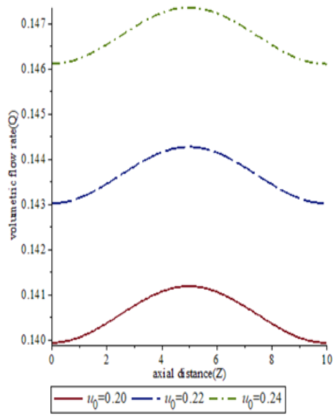

Figure 12: Variation of Volumetric Flow Rate of Unsteady MHD Blood Flow with increasing values of the Slip Velocity in the entire arterial region along the axial direction

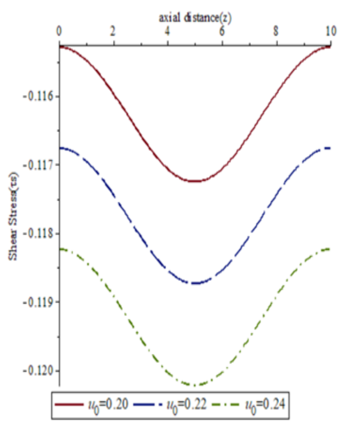

Figure 13: Variation of Shear Stress of Unsteady MHD Blood Flow with increasing values of the Slip Velocity in the entire arterial region along the axial direction

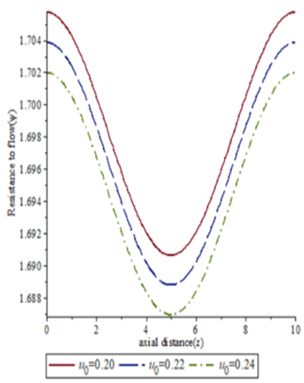

Figure 14: Variation of Resistance to Unsteady MHD Blood Flow with increasing values of the Slip Velocity in the entire arterial region along the axial direction

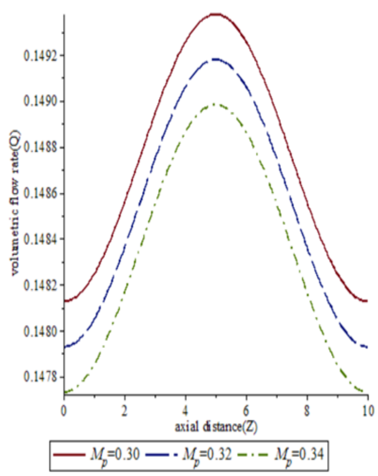

Figure 15: Variation of Volumetric Flow Rate of Unsteady MHD Blood Flow with increasing values of the Magnetic Parameter in the entire arterial region along the axial direction

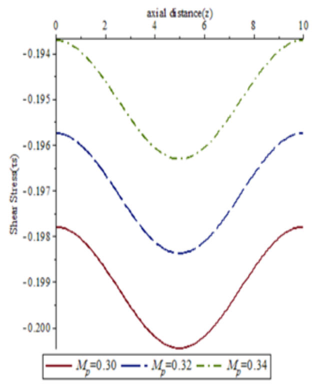

Figure 16: Variation of Shear Stress of Unsteady MHD Blood Flow with increasing values of the Magnetic Parameter in the entire arterial region along the axial direction

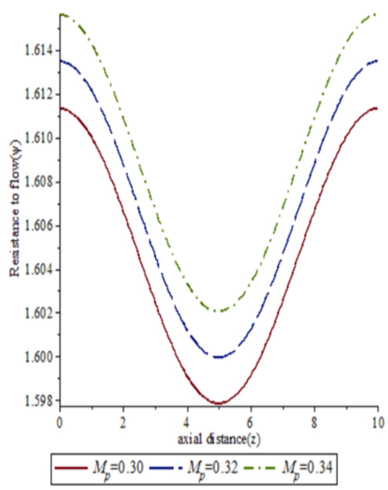

Figure 17: Variation of Resistance to Unsteady MHD Blood Flow with increasing values of the Magnetic Parameter in the entire arterial region along the axial direction

4. Results and Discussion

This study is aim at investigating the unsteady magnetohydrodynamic (MHD) blood flow through a stenosed artery with variable viscosity dependence on red blood concentration (Hematocrit) and slip velocity at the constricted artery. The impact of those parameters and other parameters on fluid velocity, volume flow rate, shear stress and resistance to flow are presented graphically.

From the graphs, it was observed that, increase in Hematocrit parameter lead to increase in shear stress and resistance to flow but decreases the flow velocity and volume flow rate. These are shown in figures 10, 11, 2 and 9 respectively. A significant decrease in fluid velocity and volume flow rate as a result of increase in hematocrit parameter as shown in figures 2 and 9 respectively occurs because increase in hematocrit parameter lead to increase in percentage volume of red blood cells and this bring about increase in density and viscosity of the blood flow relatively. Increase in density and viscosity slow down the flow velocity of blood significantly.

Also, from figure 3, increases in magnetic field parameter slightly decreases the velocity profile of the blood flow. This is because the Lorentz force which opposes the motion of the blood flow and as a result slow down the flow velocity. It is seen from figure 4 that velocity profile increases significantly with increase values of the slip velocity. This is because the slip velocity at the stenotic wall reduces the effect of induced magnetic field and viscosity and as such influencing the flow velocity positively. Other parameters that can as well influence the flow significantly are shown in figures 5, 6, 7 and 8. We observed from the figuresthat velocity profile decrease with increase values of the shear thickening as shown in figure 5 while increase with increases values of shear thinning, Reynold number and pressure gradient as shown in figures 6, 7 and 8 respectively.

Figures 2, 12, 13 and 14 illustrate the effects of slip velocity on fluid velocity, volume flow rate, shear stress and resistance to blood flow. It is found that fluid velocity and volume flow rate increase significantly with slip velocity while shear stress and resistance to flow decreases as slip velocity increases.

Figures 4, 15, 16 and 17 shows the variation of the fluid velocity, volume flow rate, shear stress and flow resistance for different values of the magnetic parameter. It is revealed from the graphs that, increase in magnetic parameter slightly lead to decrease in fluid velocity and volume flow rate as shown in figures 4 and 15 respectively. Also, shear stress and resistance to flow increases as magnetic parameter increase and these are displayed in figures 16 and 17 respectively. These happened because higher values of magnetic parameter offer more resistance to the flow while volume flow rate decreases with increases values of the magnetic parameter as illustrated in figures 17 and 15.

Other parameters that could have a positive or negative effects on the flow characteristics are Reynolds number, Pressure gradient and Hartmann number.

Conclusion: The present theoretical study investigated the unsteady magnetohydrodynamic (MHD) blood flow through a stenosed artery with variable viscosity dependence on red blood concentration (Hematocrit) and slip velocity at the constricted artery. In order to illustrate the applicability of the theoretical analysis, computational work has been carried out by simulating different values of physiological parameters available in the model. The influences of those parameters such as hematocrit parameter, slip velocity, magnetic parameter and other parameters on fluid velocity, volume flow rate, shear stress and resistance to flow are numerically evaluated using maple computer software and the results are presented graphically. The findings are highlighted below:

(1) Hematocrit parameter significantly reduces the flow velocity and flow rate but increases the wall shear stress and flow resistance.
(2) Slip velocity significantly increase the fluid velocity and volumetric flow rate but reduces the resistance to flow and shear stress.
(3) Magnetic field parameter gradually reduces the fluid velocity and volumetric flow rate but offer more resistance to the flow.
(4) Reynolds number and pressure gradient positively influence the fluid velocity while Hartmann number negatively influence the fluid velocity.
(5) Time positively influence the fluid velocity.

Finally, high red blood cell concentration (Hematocrit) which can lead to high viscosity is a risk factor in the cardiovascular disorder and as such incorporating slip velocity at the constricted artery can help to reduce blood viscosity. Also, during surgery and magnetic resonance imaging (MRI), applied magnetic field could be used to reduce the flow of blood.

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