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Forecasting Monthly Inflation Rate in Nigeria Using Causal and Invertible Autoregressive Moving Average Models

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Abstract

This paper evaluates causal and invertible ARMA models for forecasting Nigeria's inflation rate. After testing stationarity and fitting candidates, the ARMA(1, 4) model emerged as optimal. It satisfies causality, invertibility, and outperforms alternatives across loss functions.

1. Introduction and Preliminaries

The class of autoregressive moving average (ARMA) models is a combination of two distinct time series models introduced by [16] for stationary time series. ARMA models provide a unifying framework for forecasting time series. [3] popularized the use of autoregressive moving average (ARIMA) model in the literature based on a four-step iterative methodology of ARIMA modeling comprising model identification, parameter estimation, diagnostic checking and forecasting. Several authors have employed ARIMA to model and forecast inflation rate in different countries, see [4] and [5]. In [9] Akaike information criterion and Theil's U test were employed to select the best ARMA model for forecasting inflation rates in Burkina Faso. The forecast performance of the fitted ARMA models was evaluated using different forecast accuracy measures on the basis of out-of-sample forecast performance. [11] used Akaike information criterion to select the best ARMA model to forecast inflation rates in the Gambia. Forecast performance

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of the fitted ARMA models was also evaluated using different loss functions on the basis of out-of-sample forecast performance. [12] employed ARMA model to predict inflation rates in Sri Lanka using annual time series data on inflation rates in the Sri Lanka from 1960 to 2017. Forecast performance of the fitted ARMA models was evaluated using various loss functions on the basis of out-of-sample forecast performance. [10] used ARMA model to forecast inflation rates in Senegal using annual dataset from 1968 to 2017. Conventional information criteria were used for model identification and maximum likelihood method was used to estimate the parameters of the fitted ARMA model. Ljung-Box was used for model diagnotic checking. Forecast performance of the fitted ARMA models was evaluated using various loss functions on the basis of out-of-sample forecast performance. [8] used ARMA model to forecast electricity consumption in Damietta Governorate from January 2002 to December 2016. Conventional information criteria were used for model identification and maximum likelihood was used to estimate the parameters of the fitted ARMA model. Ljung-Box was used for model diagnotic checking. Performance of ARMA model was evaluated on the basis of in-sample forecast performance. [7] studied an ARMA model that best forecast hepatitis B virus infection among blood donors in Lafia, Nassarawa state of Nigeria. Partial autocorrelation function and autocorrelation function was used for model identification. Maximum likelihood method was used to estimate the parameters of the fitted and Ljung-Box was used for model diagnostic checking [2]. Forecast performance of ARMA model was evaluated on the basis of out-of-sample forecast performance. [6] used ARMA model to determine the growth rate of the exchange rate between US Dollars and a unit of British pound. [1] used ARMA and ARIMA models to forecast stock quotes based on Hurst exponent and it was shown that ARMA model outperformed ARIMA model.

2. Autoregressive Moving Average Model

A general ARMA(p, q) model is specified for modeling Nigerian inflation rate as follows:

$$X_t = \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p} + \epsilon_t - \theta_1 \epsilon_{t-1} - \cdots - \theta_q \epsilon_{t-q} \tag{1}$$

where: X_t, is inflation rate in Nigeria at time t, $\phi_1, \phi_2, ...\phi_p$ are autoregressive parameters to be estimated and $\theta_1, \theta_2, \cdots, \theta_q$ are moving average parameters to be estimated, $\epsilon_{t-1}, \epsilon_{t-2}, ...\epsilon_{t-q}$ are past errors and ϵ_t is error term at time t. The model in compact form is:

$$\phi(B) X_t = \theta(B) \epsilon_t \tag{2}$$

where the autoregressive operator is:

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - .... - \phi_p B^p \tag{3}$$

and the moving average operator is:

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - .... - \theta_q B^q \tag{4}$$

B is backshift operator such that

$$\begin{aligned} BX_t &= X_{t-1} \\ B^k X_t &= X_{t-k} \end{aligned} \tag{5}$$

For an ARMA(p, q) process to be stationary, the roots of its characteristic polynomial, defined by the autoregressive (AR) coefficients, must all lie outside the unit circle in the complex plane.

$$1 - \phi_1 z - \phi_2 z^2 - ..... - \phi_p z^p = 0 \tag{6}$$

Similarly, for an ARMA(p, q) process to be invertible, the roots of its characteristic polynomial, defined by the moving average (MA) coefficients, must all lie outside the unit circle in the complex plane

$$1 - \theta_1 z - \theta_2 z^2 - ..... - \theta_q z^q = 0 \tag{7}$$

2.1. **Causality and Invertibility of ARMA(p, q) models.** In order to verify whether the chosen optimal ARMA(p, q) model posseess unique autocovariance and autocorrelation structure, we investigated if the causality and invertibility conditions of the ARMA models are satisfied

Definition 1: A time series X_t is **Causal** if it can be expressed as

$$X_t = \epsilon_t + \psi_1 \epsilon_{t-1} + \psi_2 \epsilon_{t-2} + ... \tag{8}$$

where $\psi_1, \psi_2, ...$ are real-valued coefficients that satisfies:

$$\psi_1^2 + \psi_2^2 + ... < \infty \tag{9}$$

A time series model that can be expressed in the causal form is stationary. This implies that causality implies stationarity

Definition 2: A time series X_t is **invertible** if it can be expressed as

$$\epsilon_t = X_t - \pi_1 X_{t-1} - \pi_2 X_{t-2} - ... \tag{10}$$

where $\pi_1, \pi_2, ...$ are real-valued coefficients that satisfies:

$$\pi_1^2 + \pi_2^2 + ... < \infty \tag{11}$$

2.2. **Unit Root test for Nigeria inflation rates series.** Unit root test was carried out using augmented Dickey-Fuller (ADF) unit root test to check if there is presence of unit root in the series. This test is designed to determine whether a time series is stable around its level (which is called trend-stationary) or stable around the differences in its levels (which is called difference-stationary). To perform the ADF test, we need to choose the appropriate specification of the regression equation and the number of lagged differences to include in such specification. The choice of a particular ADF specification is determined by whether we expect the time series to have a constant mean, a linear trend, or neither. The ADF test considered in this paper has both constant mean and a linear trend. The number of truncation lag-length was determined by using information criteria, such as the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC). We consider ADF regression of the form:

$$\Delta y_t = \alpha + \beta_t + \rho y_{t-1} + \delta_1 \Delta y_{t-1} + \delta_2 \Delta y_{t-2} + ... + +\delta_p \Delta y_{t-p} + \epsilon_t \tag{12}$$

where Δy_t is the first difference of the time series y_t, α is a constant term, β_t is a linear trend term, y_{t-1} is first lag of y_t, $\Delta y_{t-1}, \Delta y_{t-2}, ..., \Delta y_{t-p}$ are first differenced lags of y_t up to p lags. The null and alternative hypotheses of augmented Dickey-Fuller (ADF) test of eqn. (8) are:

- $H_0 : \rho = 0$ - The Nigeria Inflation rate series is non-stationary
- $H_1 : \rho < 0$ - The Nigeria Inflation rate series is stationary

3. Data Analysis

The data used in this paper is monthly inflation rate in Nigeria from the period January, 2003 February, 2025 obtained from the website of Central Bank of Nigeria. Unit root test was carried out using augmented Dickey-Fuller test and the results are presented in Table 1 and Table 2.

Table 1. Unit Root Test for Inflation rate in Nigeria at level

Variable	**ADF Test Statistic**	**Critical Value at 5%**	**Decision**
Inflation rate	-1.247517	-3.430766	Accept H_0

From table 1, the ADF test statistic value is greater than critical value at 5% significance level, hence, we fail to reject the null hypothesis of unit root and we conclude that Nigeria inflation rate is non-stationary at level.

Table 2. Unit Root Test for Inflation rate in Nigeria at first difference

Variable	**ADF Test Statistic**	**Critical Value at 5%**	**Decision**
Inflation rate	-4.442780	-3.430766	Reject H_0

From table 2, the ADF test statistic value is less than test critical value at 5% significance level, hence, the null hypothesis of unit root is rejected and we conclude that Nigeria inflation rate is stationary at first difference.

3.1. **Model Identification.** Three different classes of ARMA models comprising ARMA$(p, 0)$ models, ARMA$(0, q)$ models and ARMA(p, q) models with a total of twelve different specifications of ARMA models were fitted to the Nigeria's inflation rates data. Three standard information criteria such as Akaike information criterion (AIC), Bayesian information criterion (BIC), and Hannan-Quinn information criterion (HQIC) were used for model identification and the results are presented in Table 3–5.

Table 3. Selection of optimal ARMA model among the fitted ARMA$(p, 0)$ models.

Model	**AIC**	**SIC**	**HQIC**
ARMA$(1, 0)$	4.802198	4.848774	4.821011
ARMA$(2, 0)$	4.594309	4.640885	4.613122
ARMA$(3, 0)$	5.007535	5.054111	5.026348
ARMA$(4, 0)$	5.285335	5.331911	5.304147

Table 4. Selection of optimal ARMA model among the fitted ARMA$(0, q)$ models

Model	**AIC**	**SIC**	**HQIC**
ARMA$(0, 1)$	4.746920	4.793495	4.765732
ARMA$(0, 2)$	5.018905	5.065480	5.037717
ARMA$(0, 3)$	5.121261	5.167836	5.140073
ARMA$(0, 4)$	5.341857	5.388432	5.360669

Table 5. Selection of optimal ARMA model among the fitted ARMA(p, q) model

Model	**AIC**	**SIC**	**HQIC**
ARMA$(1, 1)$	3.812484	3.874786	3.837652
ARMA$(1, 2)$	3.815346	3.867447	3.830430
ARMA$(1, 3)$	3.806840	3.869142	3.832008
ARMA$(1, 4)$	3.806626	3.868928	3.831793
ARMA$(2, 1)$	3.812897	3.875199	3.838064

From tables 3 to 5, it is obvious that ARMA$(1, 4)$ model has the smallest value of AIC, SIC and HQIC among the fitted ARMA(p, q) models. Hence, ARMA$(1, 4)$

model is chosen as the best optimal ARMA model among the class of fitted ARMA(p, q) models.

3.2. **Parameters Estimation of ARMA(p, q) model.** Following the results of model identification stage presented in tables 3 to 5, ARMA$(1, 4)$ has the least value for each of the three standard information criteria among the twelve fitted ARMA models and is therefore chosen as the best optimal model. The mathematical formulation of ARMA$(1, 4)$ model is:

$$X_t = \phi_1 X_{t-1} + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \theta_3 \epsilon_{t-3} - \theta_4 \epsilon_{t-4} \tag{13}$$

The ARMA$(1, 4)$ model with estimated parameters is expressed as:

$$X_t = 0.18 X_{t-1} + \epsilon_t - 0.53 \epsilon_{t-1} - 0.53 \epsilon_{t-2} + 0.53 \epsilon_{t-3} + 0.53 \epsilon_{t-4} \tag{14}$$

where $-1 \leq \psi \leq 1$.

Table 6. Parameters Estimation of ARMA$(1, 4)$

Model	**Coefficient**	**Standard Error**
AR(1)	0.18	0.043803
MA(1)	0.53	0.020945
MA(2)	0.53	0.038268
MA(3)	-0.53	0.040005
MA(4)	-0.53	0.041516

Table 7. Causality parameters of the fitted ARMA(p, q) models.

Model	ψ_0	ψ_1	ψ_2	ψ_3	ψ_4	$\sum_{i=0}^{4} \psi_i^2$
ARMA$(1, 1)$	1.00	0.160	0.050	0.015	0.005	1.028350
ARMA$(1, 3)$	1.00	-0.44	0.095	-0.434	-0.524	1.665557
ARMA$(1, 4)$	1.00	-0.350	-0.593	0.423	0.606	2.020314
ARMA$(2, 1)$	1.00	0.010	0.180	0.034	0.065	1.037881

It is evident from column 7 of table 7 that square summability condition is satisfied by the four ARMA models under consideration as the sum of causality parameters of each ARMA model is finite.

Table 8. Invertibility parameters of the fitted ARMA(p, q) models

Model	π_0	π_1	π_2	π_3	π_4	$\sum_{i=0}^{4} \pi_i^2$
ARMA$(1,1)$	1.00	-0.16	-0.023	-0.004	-0.001	1.026146
ARMA$(1,3)$	1.00	0.440	0.098	0.509	0.760	2.039885
ARMA$(1,4)$	1.00	0.35	0.716	-0.151	-0.610	2.030057
ARMA$(2,1)$	1.00	-0.010	-0.178	-0.028	0.027	1.033297

where $-1 \leq \pi \leq 1$.
It is evident from column 7 of table 8 that square summability condition is satisfied by the four ARMA models under consideration as the sum of invertibility parameters of each ARMA model is finite.

3.3. **Diagnostic Test for ARMA$(1, 4)$ Model.** To test for adequacy of ARMA$(1, 4)$ model, we employed Ljung-Box test. The null hypothesis of this test is that the residuals are independently distributed. This test examines the sample autocorrelation functions using test statistic $Q(k)$ which is defined as

$$Q(k) = N(N+2) \sum_{i=1}^{k} \frac{\rho_i^2}{N-i} \tag{15}$$

Where N is the sample size, ρ_i^2 is the sample autocorrelation at lag i, and k is the number of lags being tested. The null and alternative hypotheses of Ljung-Box test are:

- H_0: Residuals are white noise.
- H_1: Residuals are not white noise.

Table 9. Result of Ljung-Box Diagnostic test

Model	**Significance Level**	**p-value**
ARMA$(1,4)$	0.05	0.345

From table 9, the p-value = 0.345 is greater than 0.05, hence, we fail to reject the null hypothesis and this outcome indicates that ARMA$(1, 4)$ model fits Nigeria inflation rate data well and there is no significant evidence of remaining autocorrelation in the residuals. The residuals appear to be white noise.

3.4. **Comparative Analysis of Forecast Performance of Fitted ARMA Models.** The predictive accuracy of fitted ARMA models was evaluated using different forecast accuracy measures such as the root mean square error (RMSE), mean squared error (MSE), mean absolute error (MAE), and mean percentage absolute error (MAPE). The analysis of predictive accuracy of the fitted ARMA models is presented in table 10.

TABLE 10. Evaluation of Forecast Performance of different ARMA Models

Model	MSE	RMSE	MAE	MAPE
ARMA$(1,1)$	27.574083	5.251103	3.673451	28.673820
ARMA$(1,3)$	29.101439	5.394575	3.761362	28.599950
ARMA$(1,4)$	25.416148	5.041443	3.577085	28.445780
ARMA$(2,1)$	33.041677	5.748189	4.035507	29.736670

From table 10, it is obvious that ARMA$(1,4)$ model has the least value of each of the loss functions indicating that ARMA$(1,4)$ model has the best predictive accuracy among the four ARMA models under consideration.

Conclusion. This paper examined forecast performance of a class of causal and invertible ARMA models in forecasting Nigeria inflation rate. We employed standard information criteria, such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Hannan-Quinn information Criterion (HQIC) for model identification. The augmented Dickey-Fuller test was used to test for stationarity of inflation rate of Nigeria. Parameters of the fitted ARMA models were estimated using maximum likelihood method. Diagnostic checking for model adequacy of the fitted ARMA models was carried out using Ljung-Box test. Forecast accuracy of the fitted ARMA models were evaluated using different forecast accuracy measures such as Mean Absolute Error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE). Using various loss functions, ARMA$(1,4)$ model was identified as the best forecast model for Nigeria inflation rates. Results on estimating the dimension of a model, economic forecasts and policy, and applied economic forecasting could be sourced from [13]-[15] respectively.

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