Nigerian Journal of Mathematics and Applications
Volume **35**, (2025), 51 – 70. *Printed by Unilorin press*
©*Nig. J. Math. Appl.* *http://www.njmaman.com*

Equilibrium Analysis of Bertrand Competition Model

Abdulhakim Muhammad[1*], Aminu Muhammad[2]

Abstract

This paper offers a comprehensive examination of the major competition models in microeconomics, with a central emphasis on the Bertrand framework. In addition to Bertrand price competition, the paper also reviews different competition models to highlight how firms' strategic choices shape market outcomes. The analysis contrasts price-based and quantity-based competition and evaluates their impacts on equilibrium prices, outputs, and welfare across monopoly, duopoly, and oligopoly structures. Special attention is paid to the characteristic result of the Bertrand model, in which homogeneous-goods price competition drives the market price toward marginal cost.

1. Introduction and Preliminaries

The Bertrand model is an economic model of oligopoly in which firms producing identical (homogeneous) products compete by setting prices rather than quantities [17]. The model predicts that this price competition will drive the market price down to the level of marginal cost, resulting in zero economic profits, even if there are only two firms in the market (a duopoly).

The Bertrand model, named after the French economist Joseph Louis Francois Bertrand (1822-1900), is a model of price competition between duopoly retailers

Received: 13/08/2025, Accepted: 10/10/2025, Revised: 28/10/2025. * Corresponding author.
2015 *Mathematics Subject Classification.* 91B50, 91B55, & 91B74.
Keywords and phrases. Competition model; Equilibrium analysis; Bertrand model; Monopoly; Duopoly; Oligopoly.
[1,2] Department of Mathematical Sciences, Federal University Gusau, Zamfara, Nigeria
E-mail of the corresponding author: alhakeembunza@gmail.com
ORCID of the corresponding author: 0009-0005-2999-1412

[1]. In the Bertrand model, both parties participating in the game use price as the decision variable, while involved parties in the Cournot model take output or supply as the decision variable. The change makes the market equilibrium of the game completely different from the Cournot equilibrium. In other words, the model will lead each retailer to adopt the price under conditions of perfect competition, namely, the marginal cost pricing method. Through price competition, it shows that two retailers can easily change the amount of replenishment. However, once the seller determines the price, it may be difficult to change it. If all retailers follow it, equilibrium is established, and no one can get benefit by changing the price, making the price of the product equal to the marginal cost [5] and [10].

Economics is the social science that studies the production, distribution, and consumption of goods and services. Economics is the study of how economies function and the activities and interactions of economic agents. Microeconomics is a branch of economics that studies individual actors and markets, as well as how they interact and what happens as a result of those interactions. Examples of individual agents include homes, businesses, buyers, and sellers. Macroeconomics examines the economy as a system in which production, consumption, saving, and investment coexist, as well as factors influencing it such as the employment of labour, capital, and land resources, currency inflation, economic growth, and public policies that have an effect on these components [11].

Competition in economics refers to a situation in which various economic enterprises compete for restricted commodities by altering the four marketing mix's components: pricing, product, promotion, and site. According to traditional economic theory, competition drives businesses to create new goods, services, and technologies, giving customers more options and higher-quality goods. Prices for products are often cheaper the more options there are on the market, compared to what they would be if there was little (oligopoly) or no competition (monopoly) [20].

A multitude of factors, including those on the firm/seller side, such as the number of firms, entry hurdles, information, and the availability/accessibility of resources, affect how competitive a market is. With each customer having a willingness to pay, the quantity of customers in the market also affects competitiveness and affects the market's overall demand for the product [15].

A firm's sub-sector, or country's ability and performance to sell and supply goods and services in a given market is referred to as its competitiveness when compared to the ability and performance of other firms, subsectors, or countries in the same market. One firm is attempting to determine ways to deprive another company of market share. The Latin word "competere," which describes the competition that exists between entities in markets and industries, is where the word "competitiveness" originates [3]. It is frequently used in management discourse when discussing comparisons of regional and global economic performance.

The number of competitors, how closely they resemble one another in size, and especially how much of the industry's output is controlled by the largest firm can be used to gauge the level of competition in a given market.

Modern economic theory has focused on the many-seller limit of general equilibrium, while earlier economic research concentrated on the distinction between price and non-price-based competition. Antoine Augustin Cournot, a 19th-century economist, defined competition as an environment where pricing does not change with quantity or when the demand curve facing the firm is horizontal [14].

Resources (capital, labour, technology, and talent) tend to congregate geographically, according to empirical observation [9]. This outcome reflects the fact that businesses are intertwined with networks of suppliers, customers, and even rival businesses, giving them a competitive edge when selling their goods and services. Although arms-length market links do offer these advantages, there are occasionally externalities that result from connections among businesses in a particular region or sector (textiles, leather goods, silicon chips) that cannot be captured or promoted by markets alone. Models that highlight the benefits of networks include the "clusterization" process, "value chains," and "industrial districts."

The motivation of businesses under capitalist economic systems is to uphold and enhance their own competitiveness; this essentially applies to all business sectors. Adam Smith defined competition in his 1776 book The Wealth of Nations as the process of allocating productive resources to their most highly valued uses and promoting efficiency, a definition that quickly gained traction among liberal economists who opposed the monopolistic practices of mercantilism, the time's preeminent economic theory [19]. Prior to Cournot, Smith and other classical economists discussed price and non-price rivalry among producers in order to offer their goods on the most favorable terms to the highest-bidding purchasers, not necessarily to a sizable number of sellers or to a market that had reached final equilibrium.

Later microeconomics theory distinguished between perfect competition and imperfect competition, concluding that perfect competition is Pareto efficient while imperfect competition is not. Conversely, by Edgeworth's limit theorem, the addition of more firms to an imperfect market will cause the market to tend towards Pareto efficiency. Pareto efficiency, named after the Italian economist and political scientist Vilfredo Pareto, is an economic state where resources cannot be reallocated to make one individual better off without making at least one individual worse off [16]. It implies that resources are allocated in the most economically efficient manner; however, it does not imply equality or fairness.

1.1. **Bertrand competition.** A competition model used in economics that bears Joseph Louis François Bertrand's name is known as Bertrand competition. The interactions between businesses (sellers) that set prices and their clients (buyers) who decide on quantities at the stated pricing are described. In a critique of Antoine Augustin Cournot's book Recherches sur les Principes Math´ematiques de la Th´eorie des Richesses, in which Cournot had proposed the Cournot model, Bertrand developed his model in [2]. According to Cournot's concept, each company should choose a level of amount to sell before altering pricing to sell that level of quantity. As a result of the model's equilibrium, companies set their prices above their marginal costs, leading to the competitive price. When competitors' prices are higher than their marginal costs, Bertrand said in his review, each firm should maximize its profits by choosing a price level that is lower than those of its rivals. Francis Ysidro Edgeworth created a mathematical model of the idea in 1889, but Bertrand did not formalize the model [21].

The Bertrand model is predicated on a number of outlandish notions. For instance, it presupposes that customers desire to purchase from the company with the lowest price. This may not be the case in many areas for a number of reasons, including non-price competition, product differentiation, transportation costs, and search expenses. Would someone, for instance, go twice as far to obtain vegetables for one percent less money? Although the Bertrand model can be expanded to account for product or location differentiation, the main conclusion—that pricing is driven to marginal cost—no longer holds true. Other equilibria with regard to search costs may exist in addition to the competitive price, such as the monopoly price or even price dispersion, as in the well-known "Bargains and Rip-offs" scenario [12]. Additionally, the model disregards capacity restrictions. The "price equals marginal cost" conclusion might not hold if a single firm is unable to supply the entire market. Francis Ysidro Edgeworth initiated the investigation of this scenario, which has come to be known as the Bertrand-Edgeworth model. With capacity restrictions, a pure strategy Nash equilibrium might not exist—this is known as the Edgeworth conundrum. Huw Dixon demonstrated that there will, however, generally be a mixed-strategy Nash equilibrium [6].

Some economists have also attacked the model, claiming that it produces unfeasible results when firms have fixed costs F and, as was already indicated, constant marginal costs c. As a result, $TC = F + cQ$ is the formula for the total cost of producing Q units. According to the traditional paradigm, prices are finally pushed down to marginal cost, where businesses are no longer making any money and have no margins on infrastructure purchases. So, fixed costs cannot be recouped by companies. However, if enterprises' marginal cost curves are inclined upward, they may be able to make a profit on infra-marginal sales, which helps offset fixed expenses [7].

In the Bertrand model, where n is the number of firms in the market, there is a strong incentive to cooperate by agreeing to charge the monopolistic price, pm, and sharing the market evenly, $p = m/n$. The only Nash equilibrium of this model is not colluding and imposing marginal cost, which is the non-cooperative outcome. Collusion is therefore feasible because of the Folk Theorem when switching from a game of simultaneous moves to one of repeating moves with an unlimited horizon [4].

1.2. **Nash Equilibrium.** The most popular technique to determine the outcome of a non-cooperative game involving two or more participants in game theory is the Nash equilibrium, which bears the name of the American mathematician John Forbes Nash Jr. In a Nash equilibrium, each player is supposed to be aware of the equilibrium strategies of the other players, and changing one's own strategy offers no advantages. The concept of Nash equilibrium has been around since Cournot used it to explain how rival enterprises choose their outputs in 1838 [18].
The same concept was applied in a specific way in Antoine Augustin Cournot's oligopoly theory in 1838. According to Cournot's theory, various enterprises each decide how much production to generate in order to maximize their profits. The best output for one company is reliant upon the best outputs for the others. A pure-strategy Nash equilibrium, or Cournot equilibrium, arises when each firm's output optimizes its profits given the output of the other firms. In his examination of the stability of equilibrium, Cournot also proposed the idea of best response dynamics. However, Cournot did not define the concept broadly or apply it to any other applications [15].
The current set of strategy options represents a Nash equilibrium if each player has chosen a strategy—an action plan based on what has transpired so far in the game—and no one can raise their personal expected payoff by changing their strategy while the other players keep theirs the same.
If two players, Ali and Bilya, adopt strategies A and B, then (A, B) is a Nash equilibrium if neither Ali nor Bilya has any other viable strategies that perform better than A at maximizing their payoffs in response to the other player's choice of strategy. (A, B, C, D) in a game in which Chika and Daniel are also participants is a Nash equilibrium if A is Ali's best response to (B, C, D), Bilya's best reaction to (A, C, D), and so on [15].
Note: Nash demonstrated that every finite game has a Nash equilibrium.

2. Modelling

In this section, we will show various models in competition modelling and also use the models to solve some problems specifically using duopoly model. We will also apply oligopoly and monopoly on some problems.

2.1. **Bertrand competition.** Bertrand competition is a competition model which describes interactions among firms (sellers) that set prices and their customers (buyers) that choose quantities at the prices set. In this model, two or more firms produce a homogeneous good and compete in prices [8].

2.1.1. *Examples of Bertrand competition.*

1 Coke and Pepsi are two examples of a Bertrand oligopoly in the soft drink sector (which form a duopoly, a market with only two participants). In order to compete, both businesses alter their prices depending on a function that considers the price offered by their rival. According to this model, even even modest competition will cause prices to drop to the level of marginal costs, producing the same result as perfect competition.

2 The gasoline market serves as a common illustration of Bertrand competition. Customers (drivers) choose which gas station to fill up their vehicle after the price per litre is announced by the gas stations. The majority of customers choose the gas station with the lowest price since gasoline is a homogeneous good.

Note; here firms are selling their goods at marginal costs and thus making zero profit.

2.1.2. *Bertrand Duopoly Model.* In a duopoly model, the Bertrand competition will be a competition between two firms: firm 1 and firm 2

Two identical firms: 1 and 2 producing identical goods
Price function, $P(Q) = a - bQ$, $a, b > 0$
this implies $Q = \frac{a-P}{b}$
Marginal cost, $MC = C$
Total cost, $TC = CQ$
Average cost, $AC = C$
Therefore $MC = AC = C$
If the price for firm 1 is p_1 and the price for firm 2 is p_2
$p_1 = p_2$ implies split evenly (i.e; the two firms will share the market)
$p_1 < p_2$ implies firm 1 wins and firm 2 loses
$p_1 > p_2$ implies firm 2 wins and firm 1 loses.

2.1.3. *Nash Equilibrium (NE) strategy for Bertrand model.* If $p_1^* = p_2^* = c$, (i.e; the Nash equilibrium)

NE strategy: Here both the players are planning the best response strategy.
$p_1^* = p_2^* = c$ implies $MC = AC$
which implies $P = AC$, also the profit $P(Q) = 0$
Thus $PQ = (AC)Q$ which implies $PQ - (AC)Q = \pi$
Thus $\pi = (P - AC)Q$
Therefore $p_1^* = p_2^* = c$

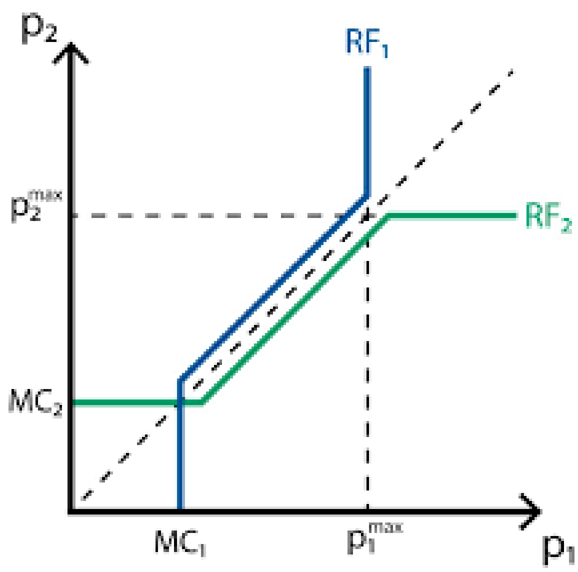

FIGURE 1. Bertrand Duopoly (from policonomics.com)

If $P < p^*$, implies $P < c$, implies $TR < TC$
Then $TR - TC < 0$
No other NE strategy
$p_1 < p_2 < c$, $p_2 < p_1 < c$
$p_1 < c < p_2$, $p_2 < c < p_1$
$c < p_1 < p_2$, $c < p_2 < p_1$
Assumptions:
If firm 1 is the low price firm, then $p_1 \leq p_2$.
Cases:

1. $c > p_1$ implies $p_1 - c < 0$ therefore p_1 losses
2. $c < p_1$ implies $p_1 = p_2 = \frac{1}{2}market$
3. $c = p_1$, see the sub cases below

sub cases for case 3:

i. $c = p_1 = p_2$, this implies the NE strategy
ii. $c = p_1 < p_2$, firm 1 will have zero profit and firm 2 will have a positive profit

If $p_1 < p_2$ and $p_1 > c$
Note: it has been proved that the only NE strategy is $p_1^* = p_2^* = c$, for $n = 2$ or $p_1^* = p_2^* = ... = p_n^* = c$, for $n \geq 2$

2.1.4. *Bertrand Oligopoly Model.* Generally, firm $i's$ individual demand function is downward sloping and a function of the price set by each firm:

$$D(p_i, p_j) = \begin{cases} D(p_i), & \text{if } p_i < p_j; \\ \frac{D(p_i)}{2}, & \text{if } p_i = p_j; \\ 0, & \text{if } p_i > p_j. \end{cases}$$

Note also that here:

(1) the market demand is continuous.
(2) the firms demand is discontinuous.
(3) the firms profit is also discontinuous.

Therefore firm i aims to maximize it's profit, as:

$\pi_i = (p_i - c)D(p_i)$.

Let p_m be the monopoly price for i and c be the marginal cost.

If $p_m = \arg max_p (p - c)D(p)$

The firms best response is:

$$R_i(p_j) = \begin{cases} p_m, & \text{if } p_j \geq p_m; \\ p_j - c, & \text{if } c < p_j < p_m; \\ c, & \text{if } p_j \leq c. \end{cases}$$

and the profit function for firm i is:

$$\pi_i = \begin{cases} (p_i - c)(\frac{a-p_i}{b}), & \text{if } p_i < p_j; \\ (\frac{p_i-c}{2})(\frac{a-p_i}{b}), & \text{if } p_i = p_j; \\ 0, & \text{if } p_i > p_j. \end{cases}$$

Note: $p_iQ - cQ = p_i(\frac{a-p_i}{b}) - c(\frac{a-p_i}{b})$

This implies, the unique NE here also is such that $p_1^* = p_2^* = ... = p_n^* = c$, for $n \geq 2$

Hence $\pi_1 = \pi_2 = ... = \pi_n = 0$

3. Results

In this chapter, we will show how the model helps in solving problems in competition modeling in a simplified approach. We will then see the difference in the market prices and quantities of the various competition models discussed, and we will also try to highlight the similarities, differences, and some advantages and disadvantages of the various models.

3.1. **Market Outcomes under Different Competition Models.** To make the main results of this study more justified and comparable, the equilibrium outcome of each competition model are presented separately before conducting a clear comparison across models. This approach highlights how strategic behaviour affects market prices and quantities under different market structures.

For instance, if we use Stackelberg model in the case where the price function $P(Q) = 100 - 2Q$ and the marginal cost $MC_1 = MC_2 = 20$, the total quantity will be $Q = 45$, market price will be $P = 55$ and the total profit will be $\pi = \pi_1 + \pi_2 = 1575$, while in the Cournot model with price function $P(Q) = 100 - 2Q$ and the marginal cost $MC_1 = MC_2 = 20$, the total quantity will be $Q = 40$, market price will be $P = 60$ and the total profit will be $\pi = \pi_1 + \pi_2 = 1600$. Showing that the Stackelberg price is lower than that of Cournot, and also the profit.

Note: The Stackelberg price is lower than the Cournot price, but greater than the Bertrand price.

Let us consider a given price function $P(Q) = 180 - 2Q$, with a marginal cost $MC = 20$ (i.e; $MC_1 = MC_2 = 20$) and relate between Stackelberg, Cournot, Bertrand and Cartel competition modelling. Let's also consider another price function given as $P(Q) = 500 - 2Q$, with marginal cost $MC_1 = MC_2 = 20$, we can also show the results in the Tables, as follows:

market price	Market quantity
60	60
140	180

TABLE 1. Stackelberg Market Structure

The Stackelberg model, characterized by leader-follower behavior, produces a lower market price and higher output than Cournot competition, but does reach the perfectly competitive outcome.

market price	Market quantity
73.33	53.33
180	160

TABLE 2. Cournot Market Structure

In the Cournot model, firms compete in quantities simultaneously. This strategic interaction leads to a higher price and lower output relative to the Stackelberg Competition.

market price	Market quantity
20	80
20	240

TABLE 3. Bertrand Market Structure

Under Bertrand competition, firms compete in prices with homogeneous products. The equilibrium price equals marginal cost, resulting in the lowest price and highest output among all the models considered.

market price	Market quantity
100	40
260	120

Table 4. Cartel Market Structure

The cartel model represents collusive behavior in which firms jointly maximize profit. This leads to the highest market price and the lowest output.
Hence, we can see that from the tables, the market price of Cartel is higher than that of all the other three models, we can also see that the market output of Cartel is lower than that of all the other three models.
Thus, considering the above tables and comparing with many other examples, we can summarize (deduce) the following facts:

(1) The higher the output the lower the price.
(2) The lower the output the higher the price.
(3) Cartel (Collusion) always assumes the highest price.
(4) Bertrand always assumes the lowest price; among the four different models.

3.2. **Comparative Analysis of Competition Models.** From Tables 1-4, the following clear ranking of market outcomes emerges:

- Market Prices:
  $P_Cartel > P_Cournot > P_Stackelberg > P_Bertrand$
- Market Quantities:
  $Q_Bertrand > Q_Stackelberg > Q_Cournot > Q_Cartel$

This comparison explicitly demonstrates that Bertrand competition yields the most competitive outcome, where price is driven down to marginal cost and total output is maximized. Despite involving only a small number of firms, Bertrand competition replicates the outcome of perfect competition due to aggressive price undercutting.
In contrast, cartel behavior leads to monopolistic outcomes, while Cournot and Stackelberg models generate intermediate results depending on whether firms compete simultaneously or sequentially.

More Examples on Bertrand Model

Note: Bertrand competition is also referred to as under cut prices:
for a duopoly model, the Bertrand competition will have: p_1 and p_2.
If $p_1 < p_2$, implies $q_1 = Q$, $q_2 = 0$,

If $p_1 > p_2$, implies $q_1 = 0$, $q_2 = Q$,
If $p_1 = p_2$, implies $q_1 = q_2 = \frac{Q}{2}$.
Then $MR \neq MC$ implies $P = MC$

First:

Let the price function be:

$$P(Q) = 500 - 2Q,$$

where $Q = q_1 + q_2$
and the marginal cost be:

$$MC = 20$$

Note: In Bertrand model, $MR \neq MC$ implies $P = MC$
Therefore $500 - 2Q = 20$
then $2Q = 500 - 20 = 480$
thus $Q = 240$
Therefore,

$$\begin{aligned} q_1 = q_2 &= \frac{Q}{2} \\ &= \frac{240}{2} = 120 \end{aligned}$$

Hence:

$$\begin{aligned} P &= 500 - 2Q \\ &= 500 - 2(240) \\ &= 20, \quad and \end{aligned}$$

$$\begin{aligned} \pi_1 = \pi_2 &= Pq_1 - cq_1 \\ &= (p - c)q_1 = 120(20 - 20) \\ &= 120(0) = 0, \quad (since \quad q_1 = q_2) \end{aligned}$$

$\Rightarrow \quad \pi = \pi_1 + \pi_2 = 0$

Note: we have seen that, in the problem above, the profit of both firms equals to zero, this is due the fact that the firms are selling their product at marginal cost.

Second:

Let the price function be:

$$P(Q) = 60 - 2Q,$$

where $Q = q_1 + q_2$
and the marginal cost be:

$$MC_1 = MC_2 = 10$$

Note: In Bertrand model, $MR \neq MC$ implies $P = MC$
Therefore $60 - 2Q = 10$
then $2Q = 60 - 10 = 50$
thus $Q = 25$

Hence:

$$\begin{aligned} P &= 60 - 2Q \\ &= 60 - 2(25) = 10, \quad and \end{aligned}$$

$$\begin{aligned} \pi &= PQ - cQ = (p - c)Q \\ &= 25(10 - 10) = 25(0) = 0 \end{aligned}$$

Note: we have seen that, in the problem above, the total profit equals to zero, so also the profit of each firm will be zero. This is due the fact that the the price is equal to the marginal cost in Bertrand competition.

Other Models

We will also apply the different models solutions we have for both Cournot and Stackelberg models to see how effective they are.

Cournot model

From the Cournot duopoly model we derive in chapter three, we have the following results: The product quantities of firm 1 and 2 are equal, i.e;

The product quantities of firm 1 and 2 are equal, hence,

$$q_1^* = q_2^* = \frac{a-c}{3b},$$

The total quantity is:

$$Q^* = q_1^* + q_2^* = \frac{2(a-c)}{3b},$$

The price function is:

$$P^* = a - bQ^* = \frac{a+2c}{3},$$

The profit function of firm 1 and 2 are equal, i.e;

$$\pi_1^* = \pi_2^* = \frac{(a-c)^2}{9b^2},$$

Thus the total profit function is:

$$\pi^* = \pi_1^* + \pi_2^* = \frac{2(a-c)^2}{9b^2}$$

Recall that, from the previous example on Cournot model, we have:

$P(Q) = 120 - Q$, the price function

$MC_1 = q_1$, the marginal cost of firm 1

$MC_2 = q_2$, the marginal cost of firm 2

Here, $a = 120$, $b = 1$ and $c_1 = q_1 = q_2$

$$\Rightarrow \quad q_1 = q_2 = \frac{a-c}{3b}$$
$$= \frac{120 - q_1}{3(1)}$$
$$= 40 - \frac{q_1}{3}$$
$$then \quad \frac{4}{3}q_1 = 40$$
$$thus \quad q_1 = 30$$

Hence:
$q_1 = q_2 = 30$, (i.e; the product quantities of firm 1 and 2 are equal)

$$Q = \frac{2(a-c)}{3b} = 2q_1 = 2q_2$$
$$= 2(30) = 60, \quad since \quad Q^* = 2q_1^* = 2q_2^*$$

$$P = \frac{a+2c}{3} = \frac{120 + 2(30)}{3}$$
$$= 60, \quad (i.e; \quad the \quad price)$$

$$\pi_1 = \pi_2 = \frac{(a-c)^2}{9b^2}$$
$$= \frac{(120-30)^2}{9(1)^2}$$
$$= 900,$$

(i.e; the profit of firm 1 and 2 are also equal)

$$thus \quad \pi = 2\frac{(a-c)^2}{9b^2}$$
$$= 2\pi_1 = 2\pi_2$$
$$= 2(900)$$
$$= 1800, \quad since \quad \pi^* = 2(\pi_1^*) = 2(\pi_2^*)$$

(i.e; the total profit)
Recall that, from the previous example on Cournot model, we have:
$P(Q) = 26 - 2Q$, i.e; the price function and
$MC_1 = MC_2 = 2$, i.e; the marginal costs of firm 1 and 2 are equal and constants.

Here, $a = 26$, $b = 2$ and $c = 2$

$$\begin{aligned}\Rightarrow \quad q_1^* = q_2^* &= \frac{a-c}{3b} \\ &= \frac{26-2}{3(2)} \\ &= 4,\end{aligned}$$

(i.e; the product quantities of firm 1 and 2 are equal)

$$\begin{aligned}thus \quad Q^* &= \frac{2(a-c)}{3b} \\ &= 2(4) \\ &= 8, \quad since \quad Q^* = 2q_1^* = 2q_2^*\end{aligned}$$

$$\begin{aligned}P^* &= \frac{a+2c}{3} \\ &= \frac{26+2(2)}{3} \\ &= 10, \quad the \quad price\end{aligned}$$

$$\begin{aligned}\pi_1^* &= \pi_2^* \\ &= \frac{(a-c)^2}{9b^2} \\ &= \frac{(26-2)^2}{9(4)} \\ &= \frac{(24)^2}{36} = 16,\end{aligned}$$

(i.e; the profit of firm 1 and 2 are also equal)

$$\begin{aligned}Thus \quad \pi^* &= \pi_1^* + \pi_2^* \\ &= \frac{2(a-c)^2}{9b^2} \\ &= 2(16) = 32, \quad since \quad \pi^* = 2(\pi_1^*) = 2(\pi_2^*)\end{aligned}$$

Stackelberg model

From the Stackelberg duopoly model we derive above, we have the following results: The product quantity of firm 1 is:

$$q_1^* = \frac{a-c}{2b},$$

the product quantity of firm 2 is:

$$q_2^* = \frac{a-c}{4b},$$

The total quantity is:

$$Q^* = q_1^* + q_2^* = \frac{3(a-c)}{4b},$$

The price function is:

$$P^* = a - bQ^* = \frac{a+3c}{4},$$

The profit of firm 1 is:

$$\pi_1^* = \frac{(a-c)^2}{8b},$$

The profit of firm 2 is:

$$\pi_2^* = \frac{(a-c)^2}{16b},$$

Thus,

$$\begin{aligned}\pi^* &= \pi_1^* + \pi_2^* \\ &= \frac{3(a-c)^2}{16b} \\ &= 3\pi_2, \quad since \quad 3\pi_2\end{aligned}$$

Recall that, from the previous example in Stackelberg model, we have:
$P(Q) = 500 - 2Q$, i.e; the price function and $MC_1 = MC_2 = 20$, i.e; the marginal costs of firm 1 and 2. Here, $a = 500$, $b = 2$ and $c = 20$
Therefore:

$$\begin{aligned}q_1^* &= \frac{a-c}{2b} \\ &= \frac{500-20}{2(2)} \\ &= 120,\end{aligned}$$

(i.e; the product quantity of firm 1)

$$\begin{aligned} q_2^* &= \frac{a-c}{4b} \\ &= \frac{500-20}{4(2)} \\ &= 60, \end{aligned}$$

(i.e; the product quantity of firm 2), thus

$$\begin{aligned} Q^* &= \frac{3(a-c)}{4b} \\ &= 3(60) \\ &= 180, \quad \textit{the} \quad \textit{total} \quad \textit{quantity} \quad Q^* = 3q_2^* \end{aligned}$$

$$\begin{aligned} P^* &= \frac{a+3c}{4} \\ &= \frac{500+3(20)}{4} \\ &= 140, \quad \textit{the} \quad \textit{price} \end{aligned}$$

$$\begin{aligned} \pi_1^* &= \frac{(a-c)^2}{8b} \\ &= \frac{(500-20)^2}{8(2)} \\ &= \frac{(480)^2}{16} = 14400, \end{aligned}$$

which is the profit of firm 1.

$$\begin{aligned} \pi_2^* &= \frac{(a-c)^2}{16b} \\ &= \frac{(500-20)^2}{16(2)} \\ &= \frac{(480)^2}{32} = 7200, \end{aligned}$$

which is the profit of firm 2. Thus

$$\begin{aligned} \pi^* = \pi_1^* + \pi_2^* & \\ &= \frac{3(a-c)^2}{16b} \\ &= 3(7200) = 21600, \end{aligned}$$

which is the total profit, $\pi^* = 3(\pi_2^*)$.

Also from the previous example, we have:
$P(Q) = 26 - 2Q$, i.e; the price function
and $MC_1 = MC_2 = 2$, i.e; the marginal costs of firm 1 and 2
Here, $a = 26$, $b = 2$ and $c = 2$
Therefore:

$$\begin{aligned} \Rightarrow \quad q_1^* &= \frac{a-c}{2b} \\ &= \frac{26-2}{2(2)} \\ &= 6, \end{aligned}$$

which is the product quantity of firm 1 and

$$\begin{aligned} q_2^* &= \frac{a-c}{4b} \\ &= \frac{26-2}{4(2)} \\ &= 3, \end{aligned}$$

which is the product quantity of firm 2

$$Q^* = \frac{3(a-c)}{4b} = 3(3) = 9, \quad the \quad total \quad quantity, \quad Q^* = 3q_2^*$$

$$\begin{aligned} P^* &= \frac{a+3c}{4} \\ &= \frac{26+3(2)}{4} \\ &= 8, \quad the \quad price \end{aligned}$$

$$\begin{aligned} \pi_1^* &= \frac{(a-c)^2}{8b} \\ &= \frac{(26-2)^2}{8(2)} \\ &= \frac{(24)^2}{16} = 36, \end{aligned}$$

which is the profit of firm 1 and

$$\begin{aligned}\pi_2^* &= \frac{(a-c)^2}{16b}\\ &= \frac{(26-2)^2}{16(2)}\\ &= \frac{(24)^2}{32} = 18,\end{aligned}$$

which is the profit of firm 2. Thus

$$\begin{aligned}\pi^* &= \pi_1^* + \pi_2^*\\ &= \frac{3(a-c)^2}{16b}\\ &= 3(18) = 54, \quad since \quad \pi^* = 3(\pi_2^*), \quad the \quad total \quad profit, \quad \pi^* = 3(\pi_2^*)\end{aligned}$$

Note: we can see that the results here and the previous ones are equal. Thus, we can apply the models to solve problems on competition in a simplified approach.

Conclusion. Competition causes commercial firms to develop new products, services and technologies, it also helps in enforcing the companies (industries) to improve the quality and quantity of their products, so that consumers can have a choice of which company will best suites their demand in the purchase of goods and services. We have seen how the models of competition differs in terms of quantities, prices and profits despite having unique prices functions and marginal costs.

Competition is mainly categorized into two parts, the perfect and imperfect competition, where the perfect competition referred to a market situation when many small companies sell individual products. Because no company is large enough to control price, each simply accepts the market price [10]. The price is determined by supply and demand. While the imperfect competition is referred to as a situation where the characteristics of an economic market do not fulfill all the necessary conditions of a perfectly competitive market [13].

Due the fact that perfect competition does not occur in real life, so we have considered the imperfect competition which consists of different models among which we discussed some of them [13]. We explained the different forms of competition, where there may be only one competing as a monopolistic competition, in the case where two companies are competing, that will be referred to as duopolistic competition (it is also a form of oligopoly) and in the case where there are few (i.e; two and above) companies are competing is also known as oligopolistic competition.

This paper explains in detail how companies compete in different forms and with different models, which depend on the market situation. For instance, in the case

of Stackelberg competition, the companies must act as a leader and a follower(s) and also move sequentially, which violates the Cournot competition, where the companies must compete as rivals to one another and also move simultaneously. Note: a company using a Stackelberg model cannot change to the Cournot model and so on. In Bertrand competition, the companies compete in prices with differentiated products rather than in Stackelberg and Cournot competition, where the companies compete in quantities with homogeneous products. In Bertrand competition, if the price p_1 of firm 1 is greater than the price p_2 of firm 2, then firm 1 loses and firm 2 wins the market and vice-versa. If the price $p_1 = p_2$, then two firms will share the market [2].
The firm with a lower price will have zero profit and the firm with a higher price will have a positive profit, i.e., after the lower price firm sold everything, then customers have no other option than to buy from the higher price firm.

Acknowledgement: The authors are grateful to Federal University Gusau for the supports they received during the compilation of this work.

Competing interests: The manuscript was read and approved by all the authors. They therefore declare that there is no conflicts of interest.

Funding: The Authors received no financial support for the research, authorship, and/or publication of this article.

References

[1] Assila B., Kobbane A. & Ben-Othman, J. (2023). Improving caching resource management: A pricing economic approach using Cournot, Bertrand, and Stackelberg game models. *International Journal of Communication Systems*. **36** (1), 3-5.

[2] Bertrand J. (1883). Review of Cournot (1838) and Leon Walras, Théorie mathématique de la richesse sociale. *Journal des Savants September*. 499-508.

[3] Boussas A. & Saber H. (2018). Competitiveness Between Meaningfulness And Meaninglessness. *International Journal of Business and Management Invention (IJBMI)* ISSN (Online). 2319-8028.

[4] Chang P. (2025). *Algorithmic collusion: theory & practice*. Doctoral dissertation, University of Oxford.

[5] Chen M. & Chen Z. L. (2015). Recent developments in dynamic pricing research: multiple products, competition, and limited demand information. *Production and Operations Management*. **24** (5), 704-731.

[6] Dixon H. D., Sbriglia P. & Somma E. (2006). Learning to collude: An experiment in convergence and equilibrium selection in oligopoly. *Research in Economics*. **60** (3), 155-167.

[7] Dwyer T. (2014). Taxation: the lost history. *American Journal of Economics and Sociology*. **73** (4), 664-988.

[8] Fisher I. (1898). Cournot and mathematical economics. *The Quarterly Journal of Economics*. **12** (2), 119-138.

[9] Florida R. (2017). The economic geography of talent. *In Economy*. 305-317. Routledge.

[10] Fudenberg D. & Maskin E. (1986). The folk theorem in repeated games with discounting or with incomplete information. *Econometrica*. **54** (3), 533-554.

[11] Goodwin N., Harris J. M., Nelson J. A., Rajkarnikar P. J., Roach B. & Torras M. (2022). *Macroeconomics in context*. Routledge. 26-30.

[12] Lahkar R. (2011). The dynamic instability of dispersed price equilibria. *Journal of Economic Theory*. **146** (5), 1796-1827.

[13] Lordkipanidze R. (2024). Theory of perfect competition- Tbilisi. Scientific Research Group to Strengthen UN and INTERPOL. **148**, 8-13. ISBN: 978-9941-8-6719-4 (PDF). https://dx.doi.org/10.13140/RG.2.2.26544.98567.

[14] Menard C. (2024). Toward the Marginal Revolution: The Foundational Status of Antoine-Augustin Cournot. *In Marginal Revolution in Economics: A Reappraisal*. 57-81. Singapore: Springer Nature Singapore.

[15] Muhammad A. & Idris L. (2025). Equilibrium Analysis of Cournot Competition Model. *International Journal of Science for Global Sustainability (IJSGS)*. **11** (3), 250-261.

[16] Muller J. Z. (2013). Capitalism and inequality: What the right and the left get wrong. *Foreign Aff.* **92** (2), 30–51. http://www.jstor.org/stable/23527455.

[17] Puu T. (2018). Bertrand. In Disequilibrium Economics: Oligopoly, Trade, and Macrodynamics. *Cham*. Springer International Publishing. 97-123.

[18] Samuelson L. (2016). Game theory in economics and beyond. *Journal of Economic Perspectives*. **30** (4), 107-130.

[19] Smith M. (2023). Adam Smith on growth and economic development. *History of Economics Review*. **86** (1), 2-15.

[20] Tepper J. (2018). *The myth of capitalism: Monopolies and the death of competition*. John Wiley & Sons.

[21] Williams R. H. (2006). *Twelve British Statisticians*. Bitingduck Press LLC.