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Efficiency of Simpson's Rule; A Case Study of Unilorin Land Area

Ayinla Ally Yeketi* and Isola Olatomiwa Oluwadamilare

Abstract

Simpson's rule, a numerical integration technique, provides an efficient method for estimating the area of an irregularly shaped large landmasses when analytical solutions are impractical. This study applies Simpson's $\frac{1}{3}$, $\frac{3}{8}$ and hybrid rules to determine the area of the University of Ilorin landmass. The hybrid method, combining the $\frac{1}{3}$ and $\frac{3}{8}$ rules, proves to be more efficient due to its flexibility with subintervals, improved accuracy and the efficient use of data points. Using the university's map with a scale of $1.2 : 100,000$, the area was calculated as approximately $15,200$ hectares across all methods. The hybrid rule yielded the smallest absolute error compared to the estimated value of $15,000$ hectares. This work demonstrates that Simpson's rule, particularly the hybrid approach, provides a practical and accurate method for land area calculations, suitable for complex geographical settings.

1. Introduction and Preliminaries

The University of Ilorin (Unilorin), Ilorin, established in 1975 [6], is one of Nigeria's leading federal universities, and is widely recognized for its expanse land area. Situated in Ilorin, Kwara State, the institution occupies a landmass of about 15,000 hectares [7], making it one of the largest universities in Africa. This

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Department of Mathematics, University of Ilorin, Nigeria
E-mails: ayinla.ay@unilorin.edu.ng
ORCID of the corresponding author: 0000-0002-9858-9102

vast land area provides space not only for academic and administrative buildings but also for staff and student residences, research farms, and future developmental projects. The size of Unilorin's landmass has become a distinguishing feature of the institution, giving it great potential for expansion, environmental conservation, and agricultural research. Its extensive geographical spread also makes it a suitable subject for mathematical and scientific studies that focus on land measurement and estimation techniques.

Simpson's rule is a numerical method used to approximate definite integrals, particularly useful when analytical integration is impossible. Named after Thomas Simpson (1710-1761), who popularized it in 1743 [9]. The method was earlier explored by some prominent mathematicians whose work laid important contributions to the formulation and establishment of the rule known as Simpson's rule. Bonaventura Cavalieri's (1598–1647) work of 1635 [8], serves as a foundational step in the development of numerical integration, with Simpson's rule as a later, more refined technique that builds on the conceptual groundwork laid by Cavalieri [4]. The work discussed Cavalieri's method of indivisibles, which approximated areas and volumes by summing infinitesimally thin slices (indivisibles). In the same vein, the development of interpolation formulas are considered as a precursor to numerical integration techniques, James Gregory's (1638–1675) [8]. Gregory's work is as well relevant in the formulation of Simpson's rule, as the use of symmetrizing division points with common sample points as that of Simpson's rule leads to Gregory's formula [1], [2] and [3]. The relevance of the works of both Cavalieri and Gregory to the formulation of the rule cannot be overemphasized, just like those of Isaac Newton, Johannes Kepler, and so on. It is widely applied in fields like engineering, physics, and economics for calculating areas, volumes, and other quantities.

This study focuses on using Simpson's $\frac{1}{3}$, $\frac{3}{8}$ and hybrid rules to estimate the area of the University of Ilorin landmass, approximately 15,000 hectares. The objective of the research is to investigate the effectiveness of Simpson's rules ($\frac{1}{3}$, $\frac{3}{8}$ and hybrid) for large landmass area calculations. The significance lies in providing a precise, practical method for land area estimation and enhancing the understanding of numerical integration applications [5].

2. Materials and Methods

This study employs Simpson's $\frac{1}{3}$, $\frac{3}{8}$ and hybrid rules to calculate the area of the University of Ilorin landmass using its map scale $(1.2 : 100,000)$. The methodology involves;

2.1. **Derivations of Simpson's Rule.**

The Simpson's rules are numerical integration methods based on polynomial interpolation [4,5]. Below is a brief overview of their derivations:

- **Simpson's $\frac{1}{3}$ Rule**: This rule approximates the integral $f(x)$ over two subintervals $[x_{i-1}, x_{i+1}]$ using a quadratic polynomial through points (x_{i-1}, y_{i-1}), (x_i, y_i) and (x_{i+1}, y_{i+1}). The integral of the quadratic over $[x_{i-1}, x_{i+1}]$ with subinterval width $h = \frac{x_{i+1}-x_{i-1}}{2}$ is :

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[(y_{i-1} + 4y_i + y_{i+1}\right]. \tag{1}$$

For n (even) subintervals over $[a, b]$, summing over pairs gives:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[(y_0 + y_n) + 2\sum_{i=2,4,\ldots,n-2} y_i + 4\sum_{i=1,3,\ldots,n-1} y_i\right]. \tag{2}$$

- **Simpson's $\frac{3}{8}$ Rule**: This rule uses a cubic polynomial to approximate $f(x)$ over three subintervals $[x_{i-1}, x_{i+2}]$ through points (x_{i-1}, y_{i-1}), (x_i, y_i), (x_{i+1}, y_{i+1}) and (x_{i+2}, y_{i+2}). Integrating the cubic over $[x_{i-1}, x_{i+2}]$ with $h = \frac{x_{i+2}-x_{i-1}}{3}$ yields:

$$\int_{x_{i-1}}^{x_{i+2}} f(x)\,dx \approx \frac{3h}{8}\left[y_{i-1} + 3y_i + 3y_{i+1} + y_{i+2}\right]. \tag{3}$$

For n (multiple of 3) subintervals, the rule is:

$$\int_a^b f(x)\,dx \approx \frac{3h}{8}\left[y_0 + 3\sum_{i=1,2,4,5,\ldots,n-2,n-1} y_i + 2\sum_{i=3,6,\ldots,n-3} y_i + y_n\right]. \tag{4}$$

- **Hybrid Rule**: For odd n, the hybrid rule applies the $\frac{3}{8}$ to the first three subintervals $[x_0, x_3]$ and the $\frac{1}{3}$ rule to the remaining $n-3$ (even) subintervals. Combining the formulas, the integral is approximated as:

$$\int_a^b f(x)\,dx \;\approx$$

$$\frac{h}{24}\left[\,9y_0 + 27y_1 + 27y_2 + 17y_3 + 32\sum_{i=4,6,\ldots,n-1} y_i + 16\sum_{i=5,7,\ldots,n-2} y_i + 8y_n\,\right]. \tag{5}$$

This rule balances accuracy and flexibility for odd subintervals, leveraging both quadratic and cubic approximations.

3. Application of the study

The methodology for applying these rules includes:

- **Dividing the Landmass:** The landmass is divided into sections with width $h = \frac{b-a}{n}$, where $a = 0$, $b = 1,858,333.333$ cm (length, as extracted from the map), and n is the number of subintervals (10 for $\frac{1}{3}$ rule, 9 for $\frac{3}{8}$ rule, 15 and for hybrid rule).
- **Measuring the Lengths:** The width is measured as $816,666.667$ cm (as extracted from the map) at regular intervals, recorded as ordinates $y_0, y_1, \ldots, y_n$.

- **Application of Simpson's Rules:**

  Simpson's $\frac{1}{3}$ Rule: For even n, the formula as established earlier is:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[(y_0+y_n)+2\sum_{i=2,4,\ldots,n-2} y_i+4\sum_{i=1,3,\ldots,n-1} y_i\right]. \tag{6}$$

  Simpson's $\frac{3}{8}$ Rule: For n, a multiple of 3, the formula as established earlier is:

$$\int_a^b f(x)\,dx \approx \frac{3h}{8}\left[(y_0+y_n)+3\sum_{i=1,2,4,5,\ldots,n-2,n-1} y_i+2\sum_{i=3,6,9,\ldots,n-3} y_i\right]. \tag{7}$$

  Hybrid Rule: For odd n, the hybrid rule applies $\frac{3}{8}$ rule to the first three sub-intervals and $\frac{1}{3}$ rule to the remaining even subintervals:

$$\int_a^b f(x)\,dx \approx \frac{h}{24}\,[\,9y_0+27y_1+27y_2+17y_3+32\sum_{i=4,6,9,\ldots,n-2} y_i \tag{8}$$

$$+16\sum_{i=5,7,9,\ldots,n-2} y_i+8y_n\,] \tag{9}$$

- **Calculating the Area:** The sum of ordinates is multiplied by the appropriate factor ($\frac{h}{3}$, $\frac{3h}{8}$, or $\frac{h}{24}$) to obtain the area in square centimeters, converted to hectares (1 hectare = $100,000,000\ cm^2$).

Data points were derived from the map's dimensions (length: 18.5833 cm, width: 8.1667 cm), scaled to real-world units ($1,858,333.333$ cm : $816,666.667$ cm).

4. Results and Discussion

The University of Ilorin landmass area was calculated using Simpson's $\frac{1}{3}$, $\frac{3}{8}$ and hybrid rules. The map's scale (1.2:100,000) was used to convert measurements to real-world units, yielding a length of $1,858,333.333$ cm and width of $816,666.667$ cm. The results are summarized in Table 1, with absolute errors computed relative to an estimated reference area of 1,500,000,000,000 cm^2 (15,000 hectares).

- **Simpson's $\frac{1}{3}$ Rule** ($n = 10$, $h = 185,833.3333$ cm):

$$\int_0^{1,858,333.333} 816,666.667\,dx \approx \frac{185,833.3333}{3}\,[816,666.667 + 4(4,083,333.335) + 2(3,266,666.668) + 816,666.667]$$

$$\int_0^{1,858,333.333} 816,666.667\,dx \approx 1,517,638,889,236.1\,\text{cm}^2$$

  Equivalent to $15,200$ hectares.

- **Simpson's $\frac{3}{8}$ Rule** ($n = 9$, $h = 206,481.4814$ cm):

$$\int_0^{1,858,333.333} 816,666.667\,dx \approx \frac{(3)(206,481.4814)}{8}\,[\,816,666.667 + 3(4,900,000.002) + 2(1,633,333.334) + 816,666.667\,]$$

$$\int_0^{1,858,333.333} 816,666.667\,dx \approx 1,517,638,888,909.4\,\text{cm}^2$$

  Equivalent to $15,200$ hectares.

- **Hybrid Rule** ($n = 15$, $h = 123,888.889$ cm):

$$\int_0^{1,858,333.333} 816,666.667\,dx \approx \frac{123,888.889}{24}\,[\,65,333,333.36 + 32(4,900,000.002) + 16(4,083,333.335) + 6,533,333.336\,]\,.$$

$$\int_0^{1,858,333.333} 816,666.667\,dx = 1,517,638,878,619.444\,\text{cm}^2$$

  Equivalent to $15,200$ hectares.

Absolute errors were calculated as $|\text{Calculated Area} - 1,500,000,000,000|$. The hybrid rule showed the smallest absolute error (17,638,878,619.444 cm^2), indicating higher accuracy. This is due to its flexibility in handling odd subintervals ($n = 15$), combining cubic ($\frac{3}{8}$ rule for first three subintervals) and quadratic ($\frac{1}{3}$

TABLE 1. Summary of Area Calculations and Absolute Errors for University of Ilorin Landmass (Reference Area: 1,500,000,000,000 cm^2 or 15,000 hectares).

Method	**Area (cm^2)**	**Absolute Error (cm^2)**
Simpson's $\frac{1}{3}$ Rule	1,517,638,889,236.1	17,638,889,236.1
Simpson's $\frac{3}{8}$ Rule	1,517,638,888,909.4	17,638,888,909.4
Hybrid Rule	1,517,638,878,619.4	17,638,878,619.4

rule for remaining subintervals) approximations, and effectively utilizing all data points without requiring additional subdivisions. The calculated areas (approximately 15,200 hectares) align with an assumed estimated range of 14.5 thousand-15.4 thousand hectares based on reported approximations for the University of Ilorin landmass.

Conclusion. Simpson's rule, particularly the hybrid approach, is highly effective for calculating the area of complex landmasses like the University of Ilorin. The hybrid rule's flexibility with odd subintervals and efficient use of data points make it superior to the $\frac{1}{3}$ and $\frac{3}{8}$ rules.

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