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A Mathematical Model of Cooperative Advertising and Subsidy Transfer Strategies

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Abstract

This work uses game theory to consider cooperative advertising in a supply chain involving a manufacturer, a retailer and a distributor. The manufacturer who is the channel leader plays a Stackelberg game with the other channel members. While the distributor is the first follower (or second leader), the retailer is the second follower. The channel structure involves a situation where only the retailer engages in advertising, while the manufacturer is indirectly involved through subsidy which may be transferred to the retailer via the distributor. It considers a four-game scenario: where there is no subsidy; where the provided subsidy is not transferred to the retailer; where the distributor personally subsidises retail advertising; and where the manufacturer's provided subsidy is transferred to the retailer. The work shows that of these four equilibria situation, the channel performs better with the transfer of provided subsidy to the retailer.

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1. Introduction

Cooperative advertising is a special arrangement in which the manufacturer reimburses the retailer for some or all the costs of advertising the manufacturer's product. Traditionally, cooperative advertising models are based on manufacturer(s)-retailer(s) relationship. But it is a known fact that a lot of manufacturers do not transact businesses directly with their retailers. Such manufacturers transfer (sell) their products to the retailers through a third part which is the distributor. By extension, incentives to the retailer must go through the distributor. Now, there is the tendency for the distributor not to transfer such fund. It is also possible for him to incentivise or even give incentive to the retailer in anticipation for a reimbursement from the manufacturer, which may never come. This work centers on these considerations.

The cooperative advertising literature can be classified into static and dynamic. Berger (1972) is considered to be the first work on cooperative advertising. He considered the concept of cooperative advertising as motivation in the form of price discount from the manufacturer to aid the retail advertising effort. Next was Dant and Berger (1996) which extended the Berger model. They investigated a franchise system in which demand is uncertain and the parties (the manufacturer and the retailer) disagree on the expected sales. These were followed by a number works on cooperative advertising which include Huang and Li (2001), Huang et al (2000), Li et al (2002), Yue et al (2006), Yang et al (2013), Xie and Neyret (2009), Kali (1998), He et al (2014).

One of the first works on dynamic cooperative advertising is Chintagunta and Jain (1992). It is based on Nerlov-Arrow model (Nerlove and Arrow (1992)). Other dynamic models in the literature include Jorgensen et al (2001), Jorgensen et al (2000), Karray and Zaccour (2005), He el al (2009), Zhang et al (2013).

All these works are based on the relationship between the manufacturer(s) and the retailer(s). This work will for the first time consider a cooperative advertising in a manufacturer-distributor-retailer channel in which subsidy is transferred by the manufacturer to the retailer through the distributor. The need for this study arises from the fact that the manufacturer does not usually/normally have direct dealing with their retailers. The transactions between the manufacturer and retailer are actually sealed through the distributor. As such advertising incentives from the manufacturer to the retailer passes through the distributor. This advertising incentive (support for retail advertising) is also referred to as subsidy or participation rate. Thus this work is based on the remittal (transfer) of such to the retailer. We will consider four scenarios:

(i) when neither of the players subsidises the retail advertising effort.

(ii) when the manufacturer provide subsidy but the distributor refuses to transfer (give) same to the retailer.

(*iii*) when the manufacturer does not provide subsidy, but the distributor considers it necessary to personally subsidise the retailer effort.
(*iv*) when the distributor transfers some or the whole subsidy to the retailer.

2. The Model

This work deals with a manufacturer-distributor-retailer supply chain in which the distributor supply the manufacturer's product to the retailer who in turn sells it to the consumers (end-users). Only the retailer is directly involved in advertising the manufacturer's product. The manufacturer subsidises the retailer advertising effort through the distributor who in turn transfers the same to the retailer. The retailer decides on his advertising effort $\beta\ \vartheta_R$, while the manufacturer and distributor decide on the participation rates β_M and β_D respectively.
To model the impact of advertising on demand we employ the increasing concave function used in Xie and Wei (2009). The aim is to incorporate the idea of continuous diminishing returns on any additional advertising spending upon saturation. This idea is also used by He et al (2011), Simon and Arndt (1980). The function is given by

$$D\left(\vartheta_R\right) = \alpha\sqrt{\vartheta_R}\ , \tag{1}$$

where α is the advertising effectiveness parameter. It represents the impact of advertising on sale (demand).
Let m_R, m_D and m_M represent the retailer, the distributor and the manufacturer's price margins respectively. Thus

$$P_R = m_R\alpha\sqrt{\vartheta_R} - \left(1-\beta_D\right)\vartheta_R \tag{2}$$

is the retailer's profit.
Similarly

$$P_D = m_D\alpha\sqrt{\vartheta_R} - \vartheta_R\beta_D + \vartheta_R\beta_M \tag{3}$$

is the distributor's profit; while

$$P_M = m_M\alpha\sqrt{\vartheta_R} - \vartheta_R\beta_M \tag{4}$$

is the manufacturer's profit.

3. The Game's Decisions Sequence

The game's order of events is as follows. The manufacturer takes the first step by informing the distributor of his margin m_M (that is the margin he is willing to sell the product to the distributor) and the participation rate β_M to be used to transfer subsidy to the retailer. Next the distributor follows by unveiling his margin m_D (the margin at which he wants to transfer the product to the retailer) and his participation rate β_D to the retailer. This is expected to be based on the subsidy from the manufacturer. Finally based on these the retailer decides on his

margin m_R and advertising effort ϑ_R. Thus armed with the manufacturer and distributor's decisions, the retailer's objective is to

$$\max P_R = m_R\alpha\sqrt{\vartheta_R} - (1-\beta_D)\,\vartheta_R \tag{5}$$

s. t. $\vartheta_R \geq 0$.
To determine the optimal value of β_D the distributor maximises the (3), that is

$$\max P_D = m_D\alpha\sqrt{\vartheta_R} - \vartheta_R\beta_D + \vartheta_R\beta_M \tag{6}$$

s. t. $0 \leq \beta_D \leq 1$.
Also, to determine the optimal value of β_M, the manufacturer maximizes (4), that is

$$\max P_M = m_M\alpha\sqrt{\vartheta_R} - \vartheta_R\beta_M \tag{7}$$

s. t. $0 \leq \beta_M \leq 1$.
Maximizing (5) with respect to ϑ_R we have that

$$\frac{\partial P_R}{\partial \vartheta_R} = \frac{m_R\alpha}{2\sqrt{\vartheta_R}} - 1 + \beta_D = 0$$

$$\Longrightarrow \quad \vartheta_R = \frac{m_R^2\alpha^2}{4\,(1-\beta_D)}\,.$$

Putting (**??**) into (6) we have

$$\max P_D = \frac{\alpha^2 m_R m_D}{2\,(1-\beta_D)} + \frac{\alpha^2 m_R^2}{2^2\,(1-\beta_D)}\;(\beta_M - \beta_D) \tag{9}$$

s. t. $0 \leq \beta_D \leq 1$.
Maximizing (**??**) with respect to β_D we have

$$\frac{\partial P_D}{\partial \beta_D} = \frac{\alpha^2 m_R m_D}{2\,(1-\beta_D)} + \frac{\alpha^2 m_R^2}{4}\left[\frac{2}{(1-\beta_D)^{\mathbf{3}}}\,(\beta_M - \beta_D) - \frac{1}{(1-\beta_D)^2}\right] = 0$$

$$\Longrightarrow \quad \beta_D = \frac{2\,(m_D + m_R\beta_M) - m_R}{2\,(m_D + m_R) - m_R}\,.$$

Using (**??**) in (7) we have

$$\max P_M \;= \frac{\alpha^2 m_R m_M}{2\,(1-\beta_D)} - \frac{\alpha^2 m_R^2}{4(1-\beta_D)^2}\beta_M \tag{}$$

s.t $0 \leq \beta_M \leq 1$.
Putting (**??**) into (**??**) we have

$$\max P_M \;= \frac{\alpha^2 m_R m_M}{2\left(1 - \frac{2(m_D + m_R\beta_M) - m_R}{2(m_D + m_R) - m_R}\right)} - \frac{\alpha^2 m_R^2}{4\left(1 - \frac{2(m_D + m_R\beta_M) - m_R}{2(m_D + m_R) - m_R}\right)^2}\beta_M \tag{12}$$

s.t $0 \leq \beta_M \leq 1$.

Maximizing with respect to β_M we have

$$\frac{\partial P_M}{\partial \beta_M} = \frac{2\alpha^2 m_R^2 m_M (2m_D + m_R)}{2(2m_R - 2m_R\beta_M)^2} - \frac{\alpha^2 m_R^2 (2m_D + m_R)^2}{4}\left[\frac{(2m_R - 2m_R\beta_M)^2 + 4\beta_M(2m_R - 2m_R\beta_M)^2 m_R}{(2m_R - 2m_R\beta_M)^4}\right] = 0$$

$$\implies \beta_M = \frac{8m_M - 2m_R(2m_D + m_R)}{8m_M + 2m_R(2m_D + m_R)}\,.$$

Using (**??**) in (**??**) we have

$$\beta_D = \frac{2\left(m_D + m_R\frac{8m_M - 2m_R(2m_D+m_R)}{8m_M+2m_R(2m_D+m_R)}\right) - m_R}{2(m_D + m_R) - m_R} = \frac{2m_D - m_R}{2m_D + m_R} + \left(\frac{2m_R}{2m_D + m_R}\right)\left(\frac{8m_M - 2m_R(2m_D + m_R)}{8m_M + 2m_R(2m_D + m_R)}\right). \quad (1$$

Thus:

Proposition 3.1 In a manufacturer-distributor-retailer supply chain where the manufacturer provides subsidy for retail advertising through the distributor, the retail advertising effort, the distributor's participation rate and manufacturer's participation rate are given by (**??**), (**??**) and (**??**) respectively.

4. No Subsidy Equlibrium

Since there is no subsidy, that is $\beta_D = \beta_M = 0$ (**??**) becomes

$$\vartheta_R = \frac{m_R^2\alpha^2}{4}\,.$$

Using (**??**) in (5) we have

$$P_R = m_R\alpha\frac{m_R\alpha}{2} - \frac{m_R^2\alpha^2}{4} = \frac{\alpha^2 m_R^2}{4}\,.$$

Putting (**??**) into (6) we have

$$P_D = m_D\alpha\frac{m_R\alpha}{2} = \frac{\alpha^2 m_D m_R}{2}\,.$$

Putting (**??**) into (7) we have

$$P_M = m_M\alpha\frac{m_R\alpha}{2} = \frac{\alpha^2 m_M m_R}{2}\,.$$

Hence: **Proposition 4.1** When the neither the manufacturer nor the retailer participates in retail advertising, the retail advertising effort is given by (**??**) and the retailer, the distributor and the manufacturer's payoffs are given by (**??**), (**??**) and (**??**) respectively.

5. Non-Transfer of Subsidy Equilibrium

Non-transfer of the manufacturer's provided subsidy to the retailer means that while $\beta_M > 0$, $\beta_D = 0$. As such (**??**) becomes

$$0 = \frac{2m_D + 2m_R\beta_D - m_R}{2m_D + 2m_R - m_R}$$

$$\implies \quad \beta_D = \frac{m_R - 2m_D}{2m_R} .$$

That is for β_M to be positive,

$$m_R > 2m_D. \tag{8}$$

Thus if the distributor is unwilling to transfer the manufacturer's provided to the retailer then the manufacturer should only provide such a subsidy if (8) holds. This means that the retailer's margin must be far greater than the distributor's margin. In other words, withholding of the subsidy (by the distributor) should only be tolerated if the retailer's margin is seen to be too high for the consumers thereby constituting a threat to the revenue of the individual players, and consequently the entire supply chain. So that giving him any additional support may amount to wasteful spending. In that case he is expected to personally engage in advertising without any support from the other players.

Observe that the distributor has first mover's advantage. Thus he decides before the retailer does. In that case $m_D > m_R$. The implication is that β_M (in (19)) is 0 (zero). That is the manufacturer will not provide any subsidy if he envisages that the distributor will not transfer the subsidy. It therefore follows that (**??**) will be

$$\vartheta_R = \frac{m_R^2\alpha^2}{4} .$$

Now (5) becomes

$$P_R = \alpha m_R\frac{\alpha m_R}{2} - \frac{\alpha^2 m_R^2}{4} = \frac{\alpha^2 m_R^2}{4} .$$

Equation (6) becomes

$$P_D = \alpha m_D\frac{\alpha m_R}{2} + \left(\frac{\alpha^2 m_R^2}{4}\right)\left(\frac{m_R - 2m_D}{2m_R}\right)$$

$$= \frac{\alpha m_D m_R}{2} + \frac{\alpha^2 m_R(m_R - 2m_D)}{8} .$$

Equation (7) becomes

$$P_M = \frac{\alpha m_M m_R}{2} - \frac{\alpha^2 m_R(m_R - 2m_D)}{8} .$$

Thus:

Proposition 5.1 Suppose the distributor fails to transfer the manufacturer's provided subsidy to the retailer, then the retail advertising effort is given by (**??**), and the retailer, the distributor and the manufacturer's payoffs are given by (**??**), (**??**) and (**??**) respectively.

6. Equilibrium Characterising the Distributor's Support for Retail Advertising in the Absence of the Manufacturer's Provision

Suppose the distributor is willing to provide subsidy despite the fact that the manufacturer does not provide any subsidy to be transferred to the retailer. This means that $\beta_M = 0$, and $\beta_D > 0$. From (**??**) we have that

$$\beta_D = \frac{2m_D - m_R}{2m_D + m_R}$$

Now, observe that β_D can only be positive if $2m_D > m_R$.
Therefore, using (**??**) in (**??**) we have that

$$\vartheta_R = \frac{\alpha^2 m_R (2m_D + m_R)}{8}.$$

Thus using (**??**) in (5) we have

$$P_R = \alpha m_R \frac{\alpha m_R}{2\left(1 - \frac{2m_D - m_R}{2m_D + m_R}\right)} - \frac{\alpha^2 m_R^2}{4\left(1 - \frac{2m_D - m_R}{2m_D + m_R}\right)^2}\left(1 - \frac{2m_D - m_R}{2m_D + m_R}\right)$$

$$= \frac{\alpha^2 m_R (2m_D + m_R)}{8}.$$

Using (**??**) and $\beta_M = 0$ in (6) we have

$$P_D = \alpha m_D \frac{\alpha m_R}{2\left(1 - \frac{2m_D - m_R}{2m_D + m_R}\right)} - \frac{\alpha^2 m_R^2}{4\left(1 - \frac{2m_D - m_R}{2m_D + m_R}\right)^2}\left(\frac{2m_D - m_R}{2m_D + m_R}\right)$$

$$= \left(\frac{\alpha (2m_D + m_R)}{4}\right)^2.$$

Also using (**??**) and $\beta_M = 0$ in (7) we have

$$P_M = \alpha m_M \frac{\alpha m_R}{2\left(1 - \frac{2m_D - m_R}{2m_D + m_R}\right)}$$

$$= \frac{\alpha^2 m_M (2m_D + m_R)}{4}.$$

Thus:

Proposition 6.1 Suppose the distributor personally supports retail advertising without the manufacturer's support, then the retail advertising effort is given by

(**??**) and the retailer, the distributor, and the manufacturer's payoff are (**??**), (**??**) and (**??**) respectively.

7. Equilibrium Characterising the Transfer of Subsidy to the Retailer

Since the distributor transfers the subsidy to the retailer, it follows that $\beta_D,\ \beta_M > 0$. Thus (**??**) becomes

$$\vartheta_R = \frac{m_R^2\alpha^2}{4\Big(1-\frac{2m_D-m_R}{2m_D+m_R}+\Big(\frac{2m_R}{2m_D+m_R}\Big)\Big(\frac{8m_M-2m_R(2m_D+m_R)}{8m_M+2m_R(2m_D+m_R)}\Big)\Big)^2}$$

$$= \left(\frac{\alpha\left(2m_D+m_R\right)\left(8m_M+2m_R\left(2m_D+m_R\right)\right)}{64m_M}\right)^2. \tag{30}$$

Using (**??**) and (**??**) in (5) we have

$$P_R = \alpha m_R\left(\frac{\alpha\left(2m_D+m_R\right)\left(8m_M+2m_R\left(2m_D+m_R\right)\right)}{64m_M}\right)$$

$$-\left(\frac{\alpha\left(2m_D+m_R\right)\left(8m_M+2m_R\left(2m_D+m_R\right)\right)}{64m_M}\right)^2\left(\frac{32m_Rm_M}{\left(2m_D+m_R\right)\left(8m_M+2m_R\left(2m_D+m_R\right)\right)}\right)$$

$$= \frac{\alpha^2m_R\left(2m_D+m_R\right)\left(8m_M+2m_R\left(2m_D+m_R\right)\right)}{128m_M}. \tag{31}$$

Now, from (**??**) and (**??**) we have observe that

$$\beta_M-\beta_D = \frac{8m_M-2m_R\left(2m_D+m_R\right)}{8m_M+2m_R\left(2m_D+m_R\right)}$$

$$-\left(\frac{2m_D-m_R}{2m_D+m_R}+\left(\frac{2m_R}{2m_D+m_R}\right)\left(\frac{8m_M-2m_R\left(2m_D+m_R\right)}{8m_M+2m_R\left(2m_D+m_R\right)}\right)\right)$$

$$= \frac{2m_R\left(m_R-2m_D\right)}{8m_M+2m_R\left(2m_D+m_R\right)}\ .$$

Thus using (**??**) and (**??**) in (6) we have

$$P_D = \alpha m_D\left(\frac{\alpha\left(2m_D+m_R\right)\left(8m_M+2m_R\left(2m_D+m_R\right)\right)}{64m_M}\right)$$

$$+\left(\frac{\alpha\left(2m_D+m_R\right)\left(8m_M+2m_R\left(2m_D+m_R\right)\right)}{64m_M}\right)^2\left(\frac{2m_R\left(m_R-2m_D\right)}{8m_M+2m_R\left(2m_D+m_R\right)}\right). \tag{33}$$

Further, using (**??**) and (**??**) in (7)

$$P_M = \alpha m_M\left(\frac{\alpha\left(2m_D+m_R\right)\left(8m_M+2m_R\left(2m_D+m_R\right)\right)}{64m_M}\right)$$

$$-\left(\frac{\alpha\left(2m_D+m_R\right)\left(8m_M+2m_R\left(2m_D+m_R\right)\right)}{64m_M}\right)^2\left(\frac{8m_M-2m_R\left(2m_D+m_R\right)}{8m_M+2m_R\left(2m_D+m_R\right)}\right)$$

$$=\frac{\alpha^2\left(2m_D+m_R\right)\left(8m_M+2m_R\left(2m_D+m_R\right)\right)}{64}$$

$$-\frac{\alpha^2(2m_D+m_R)^2\left[(8m_M)^2-\left(2m_R\left(2m_D+m_R\right)\right)^2\right]}{\left(64m_M\right)^2}. \tag{34}$$

Thus:

Proposition 7.1 If the distributor transfers the manufacturer's provided subsidy to the retailer then the retail advertising effort is given by (**??**), and the retailer, the distributor and the manufacturer's payoffs are given by (**??**), (**??**) and (**??**) respectively.

8. Numerical Illustration

Figure 1: The retailer's payoffs for the four-game Figure 2: The distributor's payoffs for the
scenario four-game scenario

Now, let the situation where the provided subsidy is transferred to the retailer; the situation where the distributor alone supports retailer; the situation where the provided subsidy is not transferred to the retailer, and the situation where no subsidy is provided, be represented by $(\beta_D, \beta_M > 0)$, $(\beta_D > 0,\ \beta_M = 0)$, $(\beta_D = 0,\ \ \beta_M > 0)$ and $(\beta_D = \beta_M = 0)$ respectively.

From Figure 1 we observe that the retailer's payoff is largest for a situation where both the manufacturer and distributor are involved in advertising. That is when the provision of subsidy is transmitted to the retailer. We also observe that the retailer's payoff is also large when the distributor provides subsidy without the manufacturer's involvement. However, this is not as large as when both the distributor and the manufacturer are involved in advertising. Further, his payoff is smallest for a situation where none of the players supports advertising or the manufacturer's provided subsidy is not transferred to the retailer. Thus the retailer's payoff is worst-off without subsidy. We observe a similar trend in Figure 2 and Figure 3. In these cases, both the distributor and manufacturer are better-off with both being involved in the provision/transfer of subsidy. Both players' payoffs are also large if the distributor personally provides subsidy without the manufacturer involvement. However, this is not as large as the case of a situation where both are involved. It is clear in all cases that the players are worst-off with none-involvement in advertising, through subsidy or non-transfer of subsidy.

Figure 3: The manufacturer's payoffs for the Figure 4: The channel payoffs for the four-

four-game scenario game scenario
Note: For easy representation in Figure 4 we let $P_{(\beta_D=\beta_M=0)} = P_{R(\beta_D=\beta_M=0)} + P_{D(\beta_D=\beta_M=0)} + P_{M(\beta_D=\beta_M=0)}$;

$$P_{(\beta_D>0,\beta_M=0)} = P_{R(\beta_D>0,\ \ \beta_M=0)} + P_{D(\beta_D>0,\ \ \beta_M=0)} + P_{M(\beta_D>0,\ \ \beta_M=0)};$$

$$P_{(\beta_D=0,\beta_M>0)} = P_{R(\beta_D=0,\ \ \beta_M>0)} + \ P_{D(\beta_D=0,\ \ \beta_M>0)} + \ P_{M(\beta_D=0,\ \ \beta_M>0)};$$

$$P_{(\beta_D,\ \beta_M>0)} = P_{R(\beta_D,\ \ \beta_M>0)} + P_{D(\beta_D,\ \ \beta_M>0)} + P_{M(\beta_D,\ \ \beta_M>0)}.$$

Considering Figure 4 we observe that just like the individual players, the channel is better-off with the involvement of the distributor and the manufacturer in advertising through subsidy. A similar reflection is obvious in a situation where only the distributor is involved in subsidy. However, this is not as large as when both the manufacturer and distributor are involved. The channel is worst-off with non-transfer of subsidy and absolute non-involvement in subsidy by the distributor and the manufacturer.

9. Concluding Remarks

This work considered cooperative advertising in a supply chain involving the manufacturer, the distributor and the retailer using game theory. It considered the distributor as an integral part of supply chain. He stands as the link between the manufacturer and the retailer where he transfers advertising support from the manufacturer to the retailer. The work observed four equilibria: a situation where there is no subsidy from both the distributor and the manufacturer; where subsidy is not remitted to the retailer; where the distributor subsidises retail advertising without provision from the manufacturer; and where both the manufacturer and the distributor are involved in provision and transfer of subsidy to the retailer.
We observed that although when the distributor personally subsidises the retail advertising effort without the manufacturer's support, the payoffs are large, the payoffs are much larger when both the manufacturer and distributor are both involved in retail advertising. Thus the individual players are better-off with the participation of both the manufacturer and distributor in retail advertising. This is also the case with the channel.
We considered the manufacturer as the channel leader playing a Stackelberg game with the other players. A situation where any of the retailer or distributor is the Stackelberg leader can provide more understanding of cooperative advertising. This is possible in situations where the retailer or distributor is influential enough on the other players. In this work only the retailer is involved in advertising. A possible extension can incorporate either the distributor's regional advertising effort or the manufacturer's national advertising effort. This can be studied with subsidy transfer options.

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